# Synchronization of Stochastic Neural Networks with Leakage Delay and Mixed Time-Varying Delays

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***Abstract*—In this paper, the synchronization of stochastic neural networks with leakage delay and mixed time-varying delays is studied. Through constructing an improved Lyapunov-Krasovskii functional, as well as employing LaSalle-type invariance principle for stochastic differential delay equations, an adaptive controller is designed to guarantee the synchronization of stochastic neural networks with leakage delay and mixed time-varying delays based on linear matrix inequality (LMI) approach. The provided conditions are expressed in terms of linear matrix inequalities, and their feasibility can be easily checked by resorting to Matlab LMI Toolbox. Moreover, the addressed system can include some famous neural network models as its special cases, which can help extend those present results. Finally, the effectiveness of the proposed method can be further illustrated with the help of a numerical example.**

***Index Terms*—Adaptive synchronization, stochastic neural network, leakage delay, lyapunov-krasovskii functional, linear matrix inequality.**

## I. Introduction

During the past few years, synchronization control of neural networks has attracted much attention due to the background of wide range applications such as associative memory, pattern recognition, image processing, and information science and so on. It is well known that time-delays may lead to some complex dynamic behaviors such as oscillation, divergence, chaos, instability, or other poor performance of the neural networks [1]. In fact, the leakage term has also great impact on the dynamical behavior of neural networks. Therefore, it is more practical to consider synchronization of coupled neural networks with discrete and distributed time-varying delays as well as leakage delay [2]-[5].

In addition, noise disturbance is a major source of instability and can lead to poor performances in neural networks. It is noted that in real nervous systems and the implementation of artificial neural networks, noise is unavoidable and should be taken into consideration in modeling. White noise brought about by some random fluctuations in the course of transmission and other probable causes has received considerable attention in the literature [6]-[10]. Recently, the stochastic neural networks attract researchers to investigate the stability and synchronization control of the stochastic neural networks [11]-[18]. However, to the best of our knowledge, the synchronization of stochastic neural networks with leakage delay and mixed time-varying delays is seldom considered.

Inspired by the above discussions, in this paper, an adaptive feedback controller is proposed for the synchronization of coupled delayed stochastic neural networks, based on LaSalle-type invariance principle for stochastic differential delay equations. To achieve the synchronization of coupled stochastic neural networks, a linear matrix inequality approach is developed.

Throughout this paper, $R^n$ and $R^{n\times n}$ denote the $n$ -dimensional Euclidean space and the set of all $n\times n$ real matrices, respectively. The superscript $T$ denotes matrix transposition, $tr(\cdot)$ denotes the trace of the corresponding matrix and $I$ denotes the identity matrix $\|\cdot\|$ stands for the Euclidean norm. $diag(\cdot)$ stands for the block diagonal matrix. $\bullet$ represents the elements below the main diagonal of a symmetric matrix. $P>0$ means that is a real symmetric positive definite matrix.

## II. Problem Formulation and Preliminaries

Consider the neural networks with time-varying delays and leakage delay terms described by the following differential equation:

$$\begin{aligned}\dot{x}(t)=&-Cx(t-\delta)+Af(x(t))+Bg(x(t-\tau_1(t)))\\&+D\int_{t-\tau_2(t)}^{t}h(x(s))ds+J,\end{aligned}\tag{1}$$

where $x(t)=(x_1(t),x_2(t),\cdots,x_n(t))^T\in R^n$ is an n-dimensional state vector of the neural networks; $x_i(t)$ is the state variable of the $i$ th neuron at time $t$. $C$ is a positive diagonal matrix, $A=(a_{ij})_{n\times n}$, $B=(b_{ij})_{n\times n}$ and $D=(d_{ij})_{n\times n}$ are, respectively, the connection weight matrix, the delayed connection weight matrices , where $n$ is the number of neurons in the indicated neural network. $f(x(t))=(f(x_1(t)),f(x_2(t)),\cdots,f(x_n(t)))^T$, $g(x(t-\tau_1(t)))$, $h(x(t))$ denote the neuron activation functions. $\delta\ge 0$, $\tau_1(t)$ and $\tau_2(t)$ are the leakage delay, the discrete time-varying delay and the distributed time-varying delay, respectively. $J$ is an external input vector.

Consider the system (1) as the drive system; the response system is as follows:

$$\begin{aligned}dy(t)=&\big[-Cy(t-\delta)+Af(y(t))+Bg(y(t-\tau_1(t)))\\&+D\int_{t-\tau_2(t)}^{t}h(y(s))ds+J+u(t)\big]dt\\&+\sigma(e(t),e(t-\delta),e(t-\tau_1(t)),e(t-\tau_2(t)))d\omega(t)\end{aligned}\tag{2}$$

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where $y(t)\in R^n$ is the state vector of the response system; $u(t)$ is the control input to be designed. The noise perturbation $\sigma_{ij}:[0,\infty)\times R^n\times R^n\times R^n\to R^{n\times n}$ is the noise intensity matrix and $\omega_j(t)(j=1,2,\cdots,n)$ is an n-dimensional Brownian motions defined on a complete probability space $(\Omega,F,P)$ with a natural filtration $\{F_t\}_{t\ge0}$ $(F_t=\sigma\{\omega(s):0\le s\le t\})$ .Take $\tau=\max\{\delta,\tau_1(t),\tau_2(t)\}$ , and $C([-\tau,0];R^n)$ denotes the family of continuous function $\phi$ from $[-\tau,0]$ to $R^n$ with the uniform norm $\|\phi\|=\sup_{-\tau\le s\le0}|\phi(s)|$. $(\Omega,F,P)$ be a complete probability space with a filtration $\{F_t\}_{t\ge0}$ satisfying the usual conditions. Denote by $C^2_{F_0}([-\tau,0];R^n)$ the family of all $F_0$ measurable, $C([-\tau,0];R^n)$ valued stochastic variables $\phi=\{\phi(s,x):-\tau\le s\le0\}$ such that $\int_{-\tau}^0 E|\phi(s)|^2ds<\infty$ , where $E[\cdot]$ stands for the correspondent expectation operator with respect to the given probability measure $P$.

Let the error $e(t)=y(t)-x(t)$, and then the error system is given as follows:

$$\begin{aligned}de(t)=\Big[&-Ce(t-\delta)+Af(e(t))+Bg(e(t-\tau_1(t)))\\&+D\int_{t-\tau_2(t)}^t h(e(s))ds+u(t)\Big]dt\\&+\sigma(e(t),e(t-\delta),e(t-\tau_1(t)),e(t-\tau_2(t)))dw(t)\end{aligned}\tag{3}$$

where $f(e(t))=f(y(t))-f(x(t))$, $g(e(t))=g(y(t))-g(x(t))$, $h(e(t))=h(y(t))-h(x(t))$. From [19] we can know that $e(\theta)=\xi(t)$ on $-\tau\le\theta\le0$ in $C^2_{F_0}([-\tau,0];R^n)$ for any given initial data and the error system (3) has a unique global solution on $t\ge0$ denoted by $e(t;\xi)$. We write $e(t;\xi)=e(t)$ for simplicity.

Throughout this paper, we make the following assumptions:

Assumption 1: There exist diagonal matrices

$$F=diag(F_1,F_2,\cdots,F_n),\ G=diag(G_1,G_2,\cdots,G_n),$$

$H$=diag ($H_1$, $H_2$, …, $H_n$), satisfying

$$0\le\frac{f_j(u)-f_j(v)}{u-v}\le F_j,\ 0\le\frac{g_j(u)-g_j(v)}{u-v}\le G_j,$$
$$0\le\frac{h_j(u)-h_j(v)}{u-v}\le H_j,$$

for all $u,v\in R,\ j=1,2,\cdots,n.$

Assumption 2: There exists positive constants $\tau_1,\tau_2,\gamma_1,\gamma_2$ such that

$$0\le\tau_1(t)\le\tau_1,0\le\tau_2(t)\le\tau_2,\dot{\tau}_1(t)\le\gamma_1<1,\dot{\tau}_2(t)\le\gamma_2<1.$$

Assumption 3: There exist positive definite matrices $Q_1$, $Q_2$, $Q_3$ and $Q_4$ such that

$$\begin{aligned}&tr[\sigma^T(x_1,x_2,x_3,x_4)\sigma(x_1,x_2,x_3,x_4)]\\&\le x_1^TQ_1x_1+x_2^TQ_2x_2+x_3^TQ_3x_3+x_4^TQ_4x_4\end{aligned}\tag{4}$$

for all $x_1,x_2,x_3,x_4\in R^n$ and $t\in R^+$.

Assumption 4: $f(0)=g(0)=h(0)\equiv0$, $\sigma(t,0,0,0,0)\equiv0$.

From Assumption 4, we have known that the system (3) admits a trivial solution $e(t,0)=0$ corresponding to the initial data $\xi=0$.

Definition 1: The two coupled neural networks (1) and (2) are said to be stochastic synchronization for almost every initial data if for every $\xi([-\tau,0];R^n)$,

$$\lim_{x\to\infty}e(t,\xi)=0$$

To proof the main result, some preliminary lemmas are presented.

*Lemma 1* [20]. Assume that system

$$dx(t)=f(t,x(t),x(t-\tau))dt+\sigma(t,x(t),x(t-\tau))dB(t),$$

there is a unique solution $x(t;\xi)$ on $t>0$ for any given initial data $\{x(\theta):-\tau\le\theta\le0\}=\xi\in C^b_{F_0}([-\tau,0];R^n)$ , moreover, both $f(x,y,t)$ and $\sigma(x,y,t)$ are locally bounded in $(x,y)$ and uniformly bounded in $t$. If there are a function $V\in C^{2,1}(R^n\times R^+;R^+)$ , $\beta\in L^1(R^+,R^+)$ and $\omega_1,\omega_2\in C(R^n;R^+)$ such that

$$LV(x,y,t)\le\beta(t)-\omega_1(x)-\omega_2(y),(x,y,t)\in R^n\times R^n\times R^+,$$
$$\omega_1(x)>\omega_2(y),\forall x\ne0,\lim_{x\to\infty}\inf_{0\le t\le\infty}V(t)=\infty\cdot$$

Then $\lim_{x\to\infty}x(t;\xi)=0$ almost surely for every $\xi\in C^b_{F_0}([-\tau,0];R^n)\cdot$

This Lemma is also called the LaShall-type invariance principle.

*Lemma 2* [19]. For any vector $x,y\in R^n$ and positive definite matrix $G$, the following matrix inequality holds

$$2xy\le x^TGx+y^TG^{-1}y.$$

*Lemma 3* [21]. For any positive definite matrix $D\in R^{n\times n}$, a scalar $\rho>0$ , vector function $\omega:[0,\rho]\to R^n$ such that the integration concerned are well defined, then

$$\rho\int_0^\rho\omega(x)D\omega(x)ds\ge\left(\int_0^\rho\omega(x)ds\right)^TD\left(\int_0^\rho\omega(x)ds\right)\cdot$$

*Lemma 4* [22]. Given matrices $\Omega_1,\Omega_2,\Omega_3$, where $\Omega_1=\Omega_1^T$ and $\Omega_2>0$, then $\Omega_1+\Omega_3^T\Omega_2^{-1}\Omega_3<0$ if and only if

$$\begin{bmatrix}\Omega_1 & \Omega_3^T\\ \Omega_3 & -\Omega_2\end{bmatrix}<0,\quad\text{or}\quad\begin{bmatrix}-\Omega_2 & \Omega_3^T\\ \Omega_3 & \Omega_1\end{bmatrix}<0.$$

Throughout the paper, we assume that $f(t)$, $g(t)$, $h(t)$, and $\sigma(\cdot)$ satisfy the usual local Lipschitz condition and linear growth condition. It is known from [12] that $e(\theta)=\xi(t)$ on $-\tau\le\theta\le0$ in $C^2_{F_0}([-\tau,0];R^n)$ for any given initial data and the error system (3) has a unique global solution on $t\ge0$ denoted by $e(t,\xi)$ .We write $e(t,\xi)=e(t)$ for simplicity. Let $C^{2,1}(R^n\times R^+;R^+)$ be the family of all nonnegative functions $V(t,e(t))$ on $R^n\times R^+$ which are continuously twice differentiable in $e(t)$ and differentiable in $t$.

$$dx(t)=f(t,x(t))dt+\sigma(t,x(t))dw(t)\,, \qquad (5)$$

For each $V\in C^{2,1}(R^n\times R^+;R^+)$, along the trajectory of the system (3), we define an operator $LV$ from $R\times R^n\times R^n$ to $R$ by

$$\begin{aligned} LV(t,x)&=V_t(t,x)+V_x(t,x)f(t,x)\\ &\quad+\frac{1}{2}trace[\sigma^T(t,x)V_{xx}\sigma(t,x)] \end{aligned}, \qquad (6)$$

where

$$V_x(t,x)=\left(\frac{\partial V(t,x)}{\partial x_1},\frac{\partial V(t,x)}{\partial x_2},\cdots,\frac{\partial V(t,x)}{\partial x_n}\right),\ V_{xx}(t,x)=\left(\frac{\partial^2 V(t,x)}{\partial x_i\partial x_j}\right)_{n\times n}.$$

## III. Main Results

In this section, the stochastic synchronization for the two coupled neural networks (1) and (2) is investigated under Assumptions 1-4.

*Theorem 1* Under Assumptions 1–4, the two coupled delayed neural networks (1) and (2) can be synchronized for almost every initial data, if there are positive definite matrices $P_2,P_3,P_4,S$ and a positive scalar $\alpha>0$, such that the following matrix inequality holds:

$$\tau_2 S\le P_4\,, \qquad (7)$$

$$\Omega=\begin{bmatrix}\Omega_{11} & -C & 0 & A & B & 0 & D\\ \ast & Q_2-P_2 & 0 & 0 & 0 & 0 & 0\\ \ast & \ast & \Omega_{33} & 0 & 0 & 0 & 0\\ \ast & \ast & \ast & -I & 0 & 0 & 0\\ \ast & \ast & \ast & \ast & -I & 0 & 0\\ \ast & \ast & \ast & \ast & \ast & \tau_2P_4-I & 0\\ \ast & \ast & \ast & \ast & \ast & \ast & -S\end{bmatrix}<0\,, \qquad (8)$$

where $\Omega_{11}=P_2+P_3+Q_1+Q_4+FF-2\alpha I+GG$, $\Omega_{33}=Q_3-(1-\gamma_1)I+HH$, the adaptive feedback controller $u(t)=k(y(t)-x(t))$, the feedback strength $k=diag(k_1,k_2,\cdots,k_n)$ is updated by the following law $\dot{k}_i=-e_i^2(t)\ (i=1,2,\cdots,n)$.

*Proof* Consider the following Lyapunov–Krasovskii function for system (3) as

$$V(t,e(t))=\sum_{i=1}^{5}V_i(t,e(t))\,, \qquad (9)$$

where $V_1(t,e(t))=e(t)^Te(t)$, $V_2(t,e(t))=\int_{t-\delta}^{t}e^T(t)P_2e(t)ds$,

$$V_3(t,e(t))=\int_{t-\tau_1(t)}^{t}e^T(s)P_3e(s)ds\,,$$

$$V_4(t,e(t))=\int_{-\tau_2(t)}^{0}\int_{t+\xi}^{t}h^T(e(s))P_4h(e(s))dsd\xi\,,$$

$$V_5(t,e(t))=\sum_{i=1}^{n}(k_i+\alpha)^2\,.$$

Then it follows from (4) and (5) that

$$\begin{aligned} &LV_1(t,e(t))\\ &=2e(t)^T[-Ce(t-\delta)+Af(e(t))+Bg(e(t-\tau_1(t)))\\ &\quad+D\int_{t-\tau_2(t)}^{t}h(e(s))ds+ke(t)]+tr[\sigma^T(e(t),e(t-\delta),e(t-\tau_1(t)),\\ &\quad e(t-\tau_2(t)))\sigma^T(e(t),e(t-\delta),e(t-\tau_1(t)),e(t-\tau_2(t)))]\\ &=-e(t)^T2Ce(t-\delta)+e(t)^T2Af(e(t))+e(t)^T2Bg(e(t-\tau_1(t)))\\ &\quad+e(t)^T2D\int_{t-\tau_2(t)}^{t}h(e(s))ds+tr[\sigma^T(e(t),e(t-\delta),e(t-\tau_1(t)),\\ &\quad e(t-\tau_2(t)))\sigma^T(e(t),e(t-\delta),e(t-\tau_1(t)),e(t-\tau_2(t)))]\\ &\quad+2e(t)^Tke(t). \end{aligned} \qquad (10)$$

From Lemma 2, we have

$$\begin{aligned} 2e^T(t)D\int_{t-\tau_2(t)}^{t}h(e(s))ds&\le e^T(t)[DS^{-1}D^T]e(t)\\ &\quad+\left(\int_{t-\tau_2(t)}^{t}h(e(s))ds\right)^TS\left(\int_{t-\tau_2(t)}^{t}h(e(s))ds\right), \end{aligned} \qquad (11)$$

where $S$ is a positive definite matrix. Using Lemma 3, we have

$$\begin{aligned} &\left(\int_{t-\tau_2(t)}^{t}h(e(s))ds\right)^TS\left(\int_{t-\tau_2(t)}^{t}h(e(s))ds\right)\\ &\le\tau_2(t)\int_{t-\tau_2(t)}^{t}h^T(e(s))Sh(e(s))ds\\ &\le\int_{t-\tau_2(t)}^{t}h^T(e(s))(\tau_2S)h(e(s))ds \end{aligned} \qquad (12)$$

It follows from Assumption 3 and (8) that

$$\begin{aligned} &tr[\sigma^T(t,e(t),e(t-\delta),e(t-\tau_1(t)),e(t-\tau_2(t)))\\ &\quad\sigma(t,e(t),e(t-\delta),e(t-\tau_1(t)),e(t-\tau_2(t)))]\\ &\le e(t)^TQ_1e(t)+e(t-\delta)^TQ_2e(t-\delta)+e(t-\tau_1(t))^TQ_3e(t-\tau_1(t))\\ &\quad+e(t-\tau_2(t))^TQ_4e(t-\tau_2(t))) \end{aligned} \qquad (13)$$

By Ito's differential formula [21], we could infer that

$$LV_2(t,e(t))=e(t)^TP_2e(t)-e(t-\delta)^TP_2e(t-\delta)\,, \qquad (14)$$

$$\begin{aligned} LV_3(t,e(t))&=e(t)^TP_3e(t)-e^T(t-\tau_1(t))[(1-\dot{\tau}_1(t))P_3]e(t-\tau_1(t))\\ &\le e(t)^TP_3e(t)-(1-\gamma_1)e^T(t-\tau_1(t))P_3e(t-\tau_1(t)), \end{aligned} \qquad (15)$$

$$\begin{aligned} &LV_4(t,e(t))\\ &=\tau_2(t)h^T(e(t))P_4h(e(t))-\int_{t-\tau_2(t)}^{t}h^T(e(s))P_4h(e(s))ds\\ &\le\tau_2h^T(e(t))P_4h(e(t))-\int_{t-\tau_2(t)}^{t}h^T(e(s))P_4h(e(s))ds, \end{aligned} \qquad (16)$$

$$LV_5(t,e(t))=-2\sum_{i=1}^{n}(k_i+\alpha)e_i^2(t)\,. \qquad (17)$$

Furthermore, from Assumption 1, 2, we have

$$e(t)^TFFe(t)-f^T(e(t))f(e(t))\ge 0\,, \qquad (18)$$

$$e(t)^T HHe(t) - h^T(e(t))h(e(t)) \geq 0 \ , \tag{19}$$

$$e^T(t-\tau_1(t))GGe(t-\tau_1(t)) - g^T(e(t-\tau_1(t)))g(e(t-\tau_1(t))) \geq 0. \tag{20}$$

Substituting inequalities (9)–(20) into (8), it can be derived that

$$\begin{aligned}
&LV(t,e(t)) \\
&\leq -2e(t)^T Ce(t-\delta) + 2e(t)^T Af(e(t)) + 2e(t)^T Bg(e(t-\tau_1(t))) \\
&\quad + e^T(t)[DS^{-1}D^T]e(t) + \int_{t-\tau_2(t)}^{t} h^T(e(s))(\tau_2 S)h(e(s))ds \\
&\quad + e(t)^T Q_1 e(t) + e(t-\delta)^T Q_2 e(t-\delta) + e(t-\tau_1(t))^T Q_3 e(t-\tau_1(t)) \\
&\quad + e(t-\tau_2(t))^T Q_4 e(t-\tau_2(t))) + e(t)^T ke(t) + e(t)^T P_2 e(t) \\
&\quad - e(t-\delta)^T P_2 e(t-\delta) + e(t)^T P_3 e(t) - (1-\gamma_1)e^T(t-\tau_1(t))P_3 e(t-\tau_1(t)) \\
&\quad + \tau_2 h^T(e(t))P_4 h(e(t)) - \int_{t-\tau_2(t)}^{t} h^T(e(s))P_4 h(e(s))ds \\
&\quad - 2\sum_{i=1}^{n}(k_i+\alpha)e_i^2(t) + e(t)^T FFe(t) - f^T(e(t))f(e(t)) + e(t)^T HHe(t) \\
&\quad - h^T(e(t))h(e(t)) + e^T(t-\tau_1(t))GGe(t-\tau_1(t)) \\
&\quad - g^T(e(t-\tau_1(t)))g(e(t-\tau_1(t))) + e(t)^T Q_4 e(t) - e(t)^T Q_4 e(t) \\
&\leq e^T(t)[DS^{-1}D^T + P_2 + P_3 + Q_1 + Q_4 + FF + HH - 2\alpha I]e(t) \\
&\quad + e(t-\delta)^T[Q_2 - P_2]e(t-\delta) \\
&\quad + e(t-\tau_1(t))^T[Q_3 - (1-\gamma_1)P_3 + GG]e(t-\tau_1(t)) \\
&\quad + 2e(t)^T Af(e(t)) + 2e(t)^T{}_1 Bg(e(t-\tau_1(t))) \\
&\quad - e^T(t)Q_4 e(t) + e(t-\tau_2(t))^T Q_4 e(t-\tau_2(t))) \\
&\quad - f^T(e(t))f(e(t)) - g^T(e(t-\tau_1(t)))g(e(t-\tau_1(t))) \\
&\quad + h^T(e(t))[\tau_2 P_4 - I]h(e(t)) - 2e(t)^T C(e(t-\delta)) \\
&= \eta^T \prod \eta - e(t)^T Q_4 e(t) + e(t-\tau_2(t))^T Q_4 e(t-\tau_2(t)),
\end{aligned} \tag{21}$$

where

$$\eta = [e^T(t), e^T(t-\delta), e^T(t-\tau_1(t)), f^T(e(t)), g^T(e(t-\tau_1(t))), h^T(e(t))] \cdot$$

$$\Pi = \begin{bmatrix} \Pi_{11} & -C & 0 & A & B & 0 \\ * & Q_2 - P_2 & 0 & 0 & 0 & 0 \\ * & * & \Pi_{33} & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & \tau_2 P_4 - I \end{bmatrix}$$

$$\prod_{11} = DS^{-1}D^T + Q_1 + P_2 + P_3 + FF - 2\alpha I + Q_4 + HH \ ,$$

$$\prod_{33} = Q_3 - (1-\gamma_1)P_3 + GG \cdot$$

The constant $\alpha$ plays an important role in making the matrix $\prod$ negative definite. Let $-\lambda_1$ denote the largest eigenvalue of the matrix $\prod$. From (14), we have

$$\begin{aligned}
LV(t,e(t)) &\leq -e(t)^T(Q_4 + \lambda_1 I)e(t) \\
&\quad + e(t-\tau_2(t))^T(Q_4 - \lambda_1 I)e(t-\tau_2(t)) \\
&= -\omega_1(e(t)) + \omega_2(e(t-\tau_2(t))).
\end{aligned} \tag{22}$$

It can be seen that $\omega_1(e(t)) > \omega_2(e(t-\tau_2(t)))$ for any $e(t) \neq 0$. Therefore, by Lemma 1 LaSalle-type invariance principle for the stochastic differential delay equations, we can conclude that the neural networks (1) and (2) can be synchronized for almost every initial data. The proof is complete.

*Remark 1* Theorem 1 gives a sufficient condition to prove that the two coupled neural networks (1) and (2) can be synchronized for almost every initial data. This condition only depends on delay constants $\tau_2$ and $\gamma_1$.

*Theorem 2* Under Assumptions 1–4, the two coupled delayed neural networks (1) and (2) without the leakage delay ($\delta$=0) can be synchronized for almost every initial data, if there are positive diagonal matrices $P_2, P_3, S$ ,and a positive scalar $\alpha > 0$, such that the following matrix inequality holds:

$$\tau_2 S \leq P_3 \ , \tag{23}$$

$$\Omega = \begin{bmatrix} \Omega_{11} & 0 & A & B & 0 \\ * & \Omega_{22} & 0 & 0 & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & \tau_2 P_3 - I \end{bmatrix} < 0, \tag{24}$$

where $\Omega_{11} = -2C + P_2 + Q_1 + Q_3 + FF + HH - 2\alpha I$ , $\Omega_{22} = Q_2 - (1-\gamma_1)P_2 + GG$ , the adaptive feedback controller $u(t) = k(y(t) - x(t))$ , the feedback strength $k = diag(k_1, k_2, \cdots, k_n)$ is updated by the following law $\dot{k}_i = -e_i^2(t) \ \ (i=1,2,\cdots,n)$.

*Proof*: Consider the following Lyapunov–Krasovskii function for system (3) as

$$V(t,e(t)) = \sum_{i=1}^{4} V_i(t,e(t)) \ , \tag{25}$$

where

$$V_1(t,e(t)) = e(t)^T e(t) \ , V_2(t,e(t)) = \int_{t-\tau_1(t)}^{t} e^T(s)P_2 e(s)ds \ ,$$

$$V_3(t,e(t)) = \int_{-\tau_2(t)}^{0}\int_{t+\xi}^{t} h^T(e(s))\mathrm{P}_3 h(e(s))dsd\xi \ ,$$

$$V_4(t,e(t)) = \sum_{i=1}^{n}(k_i + \alpha)^2$$

Then along the same line of Theorem 1, we can obtain

$$\Pi = \begin{bmatrix} \Pi_{11} & 0 & A & B & 0 & D \\ * & \Pi_{22} & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & \tau_2 P_3 - I & 0 \\ * & * & * & * & * & -S \end{bmatrix}, \tag{26}$$

where

$$\Pi_{11} = -2C + P_2 + Q_1 + Q_3 + FF + HH - 2\alpha I + DS^{-1}D^T \ ,$$

$$\Pi_{22} = Q_2 - (1-\gamma_1)P_2 + GG \cdot$$

From Lemma 4, (26) holds if and only if (24) holds. This completes the proof.

*Remark 2:* When $\sigma(\cdot)$, compared with the theorems of [3] and [12], Theorem 2 of this paper is easier to achieve the synchronization of stochastic neural networks with leakage delay and mixed time-varying delays.

## IV. Numerical Example

In the following, we give some numerical simulations to illustrate the results above. Consider the drive neural network (1) with $\delta$=0.5,

$$C=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},\ A=\begin{pmatrix}1 & 0.5\\ -0.5 & 1.5\end{pmatrix},$$

$$B=\begin{pmatrix}-2.0 & -0.1\\ -0.2 & -2.7\end{pmatrix},\ D=\begin{pmatrix}-1.3 & 0.5\\ 0.1 & -0.5\end{pmatrix},$$

$$\tau_1(t)=0.5|\sin t|,\ J=0,\ \tau_2(t)=0.1|\cos 2t|.$$

$$f(x(t))=g(x(t))=h(x(t))=\tanh(x(t)),$$

with initial values $x_1(s)=0.01, x_2(s)=0.1$, $\forall s\in[-0.5,0]$.

The response neural network (2) with $\sigma(t,e(t),e(t-\delta),e(t-\tau_1(t)),e(t-\tau_2(t)))$

$$=\begin{pmatrix}0.3e(t)+0.2e(t-\delta) & 0\\ 0 & 0.1e(t-\tau_1(t))+0.4e(t-\tau_2(t))\end{pmatrix},$$

$$\gamma_1=0.5,\ Q_1=\begin{pmatrix}0.1 & 0\\ 0 & 0.5\end{pmatrix},\ Q_2=\begin{pmatrix}0.05 & 0\\ 0 & 0.1\end{pmatrix},$$

$$Q_3=\begin{pmatrix}0.2 & 0\\ 0 & 0.02\end{pmatrix},\ Q_4=\begin{pmatrix}0.02 & 0\\ 0 & 0.1\end{pmatrix},\ F=\begin{pmatrix}0.5 & 0\\ 0 & 0.01\end{pmatrix},$$

$G=\begin{pmatrix}0.1 & 0\\ 0 & 0.05\end{pmatrix}$ , $G=\begin{pmatrix}0.1 & 0\\ 0 & 0.5\end{pmatrix}$ with initial values, $y_1(s)=0.9$ , $y_2(s)=0.6$ , $\forall s\in[-0.5,0]$. let $\alpha=30$ , using LMI toolbox in Matlab, we can obtain the following feasible solutions to LMIs (6)–(7) :

$$P_2=\begin{pmatrix}22.5515 & -0.0428\\ -0.0428 & 21.7157\end{pmatrix},\ P_3=\begin{pmatrix}22.1673 & -0.0475\\ -0.0475 & 21.2285\end{pmatrix},$$

$$P_4=\begin{pmatrix}1.9045 & 0.0026\\ 0.0026 & 1.9091\end{pmatrix},\ S=\begin{pmatrix}3.4042 & 0.0320\\ 0.0320 & 3.4597\end{pmatrix}.$$

According to Theorem 1, the response system and the drive system with the controller $u(t)$ can be globally exponentially square-mean synchronized. Moreover, according to Theorem 2, we see that the system (1) and (2) without leakage delay term are globally synchronized

## V. Conclusion

In this paper, an adaptive feedback controller is presented to investigate the synchronization problem of stochastic neural networks with leakage delay and mixed time-varying delays. To guarantee the response system can be synchronized with the drive system Lyapunov stability theory, stochastic analysis theory, LaSalle-type invariance principle for stochastic differential delay equations are used. The synchronization criteria are easily verified. To the end, a numerical simulation is carried out to illustrate the effectiveness of the obtained results.

## References


[1] J. Park and O. Kwon, "On improved delay-dependent criterion for global stability of bidirectional associative memory neural networks with time-varying delays," *Appl. Math. Compute*, vol. 199, no. 2, pp. 435-446, Jun. 2008.

[2] X. Nie and J. Cao, "Multistability of competitive neural networks with time-varying and distributed delays," *Nonlinear Anal.: Real World Appl.*, vol. 10, no. 2, pp. 928-942, Apr. 2009.

[3] Q. Song and J. Cao, "Synchronization of nonidentical chaotic neural networks with leakage delay and mixed time-varying delays," *Advances in Difference Equations*, Jun. 2011, 2011:16.

[4] P. Balasubramaniam, V. Vembarasan, and R. Rakkiyappan, "Leakage delays in T-S fuzzy cellular neural networks," *Neural Process. Lett.*, vol. 33, no. 2, pp. 111-136, Apr. 2011.

[5] P. Balasubramaniam, G. Nagamani, and R. Rakkiyappan, "Passivity analysis for neural networks of neutral type with Markovian jumping parameters and time delay in the leakage term," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 16, no. 11, pp. 4422-4437, Nov. 2011.

[6] S. Hassan and A. Aria, "Adaptive synchronization of two chaotic systems with stochastic unknown parameters," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 14, no. 2, pp. 508-519, Feb. 2009.

[7] J. Lu, D. W. C. Ho, and Z. Wang, "Pinning stabilization of linearly coupled stochastic neural networks via minimum number of controllers," *IEEE Trans. Neural Netw.*, vol. 20, no. 10, Oct. 2009.

[8] Y. Sun, J. Cao, and Z. Wang, "Exponential synchronization of stochastic perturbed chaotic delayed neural networks," *Neurocomputing*, vol. 70, no. 13-15, pp. 2477-2485, Aug. 2007.

[9] Z. Wang, Y. Liu, M. Li, and X. Liu, "Stability analysis for stochastic Cohen-Grossberg neural networks with mixed time delays," *IEEE Trans. Neural Netw.*,vol. 17, no. 3, pp. 814-820, May 2006.

[10] W. Yu and J. Cao, "Synchronization control of stochastic delayed neural networks," *Physica A: Statistical Mechanics and its Applications*, vol. 373, no. 1, pp. 252-260, Jan. 2007.

[11] H. Gu, "Adaptive synchronization for competitive neural networks with different time scales and stochastic perturbation," *Neurocomputing*, vol. 73, no. 1-3, pp. 350-356, Dec. 2009.

[12] Q. Gan, R. Xu, and X. Kang, "Synchronization of chaotic neural networks with mixed time delays," *Commun Nonlinear Sci Numer. Simul.*, vol. 16, no. 2, pp. 966-974, Feb. 2011.

[13] Q. Gan, "Adaptive synchronization of stochastic neural networks with mixed time delays and reaction-diffusion terms," *Nonlinear Dyn.*, vol. 69, no. 4, pp. 2207-2219, Sep. 2012.

[14] Q. Zhu and J. Cao, "Exponential stability analysis of stochastic reaction-diffusion Cohen-Grossberg neural networks with mixed delays," *Neurocomputing*, vol. 74, no. 17, pp. 3084-3091, Oct. 2011.

[15] T. Ma and J. Fu, "On the exponential synchronization of stochastic impulsive chaotic delayed neural networks," *Neurocomputing*, vol. 74, no. 5, pp. 857-862, Feb. 2011.

[16] C. Zheng, F. Zhou, and Z. Wang, "Stochastic exponential synchronization of jumping chaotic neural networks with mixed delays," *Commun. Nonlinear Sci. Numer. Simulat.*, vol. 17, no. 3, pp. 1273-1291, Mar. 2012.

[17] X. Li and X. Fu, "Synchronization of chaotic delayed neural networks with impulsive and stochastic perturbations," *Commun. Nonlinear Sci. Numer. Simulat.*, vol. 16, no. 2, pp. 885-894, Feb. 2011.

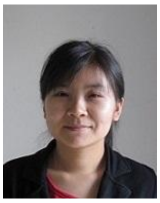
[18] X. Mao, *Stochastic Differential Equation and Application*, Horwood Publishing, Chichester, 1997.

[19] M. A. Cohen and S. Grossberg, "Absolute stability of global pattern formation and parallel memory storage by competitive neural networks," *IEEE Trans. Syst. Man Cybern. B*, vol. SMC-13, no. 5, Sep./Oct. 1983.

[20] X. Mao, "A note on the LaSalle-type theorems for stochastic differential delay equations," *J. Math. Anal. Appl.*, vol. 268, pp. 125-142, 2002.

[21] X. Liao, Y. Liu, S. Guo, and H. Mai, "Asymptotic stability of delayed neural networks: A descriptor system approach," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 14, no. 7, pp. 3120-3133, Jul. 2009.

[22] Q. Zhu and J. Cao, "Adaptive synchronization under almost every initial data for stochastic neural networks with time-varying delays and distributed delays," *Commun. Nonlinear Sci. Numer. Simulat.*, vol. 16, no. 4, pp. 2139-2159, Apr. 2011.



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