Complete Circuit Level Random Variation Models of Nanoscale MOS Performance

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***Abstract*— In this research, the complete analytical circuit level models of random variations in large and small signal parameters of any nanoscale MOS transistor have been proposed. Both triode and saturation regions have been explored. This research has been performed based upon the up to dated nanoscale regime MOS equations with the inclusion of all leading sources of variations instead of only threshold voltage variation. The proposed models have been verified at the nanometer level by using the Monte Carlo SPICE simulations and the Kolmogorov-Smirnof goodness of fit tests. These models are very accurate since they can fit the Monte Carlo based distribution with as high as 99% confidence. Obviously, they eliminate the gap between the circuit level and physical level design since the mismatches in the circuit level parameters can be now analytically formulated in terms of the physical level ones. So, the physical level causes and their relationships with the resulting circuit level mismatches can be revealed. Beside, the proposed models can also be expected to be the potential mathematical foundations for implementating the Electronic CAD cell libraries of the nanoscale MOS transistors. Hence, these models have been found to be efficient for the statistical/variability aware design of various CMOS analog/mixed signal circuits and systems in the nanoscale regime.**

***Index Terms*— Nanoscale, CMOS, analog, mixed signal, circuit level, physical level, statistical design, variability aware design, circuit, system.**

I. Introduction

Random variations in MOS performances play a very important role in the statistical/variability aware design of CMOS analog/mixed signal circuits and systems. These variations produce the random mismatches in MOS physical parameters such as threshold voltage, gate oxide capacitance and mobility etc., which are traditionally modelled as normally distributed random variable with zero mean while many formulas have been proposed for the variances for example those in [1], [2], [3], [4] and [5] etc.

For the analysis/design simplicity, random variations in MOS circuit level parameters have been studied and modelled as proposed in many previous researches for example [5], [6] and [7] etc. In [6], the percentage of variations in drain current (I_d) which is a key large signal circuit level parameter and the related small signal parameters have been modelled as the functions of the percentage of variation in threshold voltage. However, the model derivation basis adopted in [6] relies on the simple sensitivity analysis which only the standard deviations of the related parameters involved, where as the corresponding distribution functions have not been discussed. Furthermore, both [5] and [6] have been performed based upon the conventional MOS equations which are invalid in the nanoscale regime.

In [7], the models of random variation in I_d have been proposed as the analytical expressions and distribution functions not just the standard deviations. Obviously, means, variances, moments and other statistical parameters can be determined by using these models. Both triode and saturation regions have been considered. Unlike [5] and [6], this research has been performed based upon the up to dated nanoscale regime MOS equations [8]. However, the models in [7] are incomplete since it take only the threshold voltage variation into account where as the other leading variations are neglected. Furthermore, the similar models for the related small signal parameters have not been derived even though they are also necessary.

Hence, the complete analytical models of random variations in nanoscale MOS transistor large and small signal parameters which are of the circuit level, have been proposed in this research with the improvement as the inclusion of the other leading sources of variations apart from the threshold voltage variation. The proposed models have been verified at the nanometer level by using the Monte Carlo SPICE simulations and the Kolmogorov-Smirnof goodness of fit tests. These models are very accurate since they can fit the Monte Carlo based distribution with as high as 99% confidence.

Obviously, the proposed models can effectively eliminate the gap between the circuit level and physical level design since the mismatches in the circuit level parameters can be now analytically formulated in terms of the physical level ones. As such, the mismatches in the key parameters of any circuits and systems which are traditionally expressed at the circuit level can be now analytically expressed in terms of the physical level parameters. So, the physical causes and their relationships to the corresponding circuit level random mismatches can be revealed. Furthermore, the models can be expected to be the potential mathematical foundations for implementating the Electronic CAD (ECAD) cell libraries of the nanoscale MOS transistors. Hence, the proposed models have been found to be efficient for the statistical/variability aware design of various CMOS analog/mixed signal circuits and systems in the nanoscale regime

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II. THE PROPOSED MODELS

In this section, the proposed models for both triode and saturation regions will be discussed respectively. Before proceed further, it should be mentioned here that the Pelgrom's model for the variation in threshold voltage [1] which has been adopted in [7], is also adopted in this research. At the nanoscale regime, the devices are closely spaced. Hence, the distribution function of the random threshold voltage variation (ΔV_t) can be given by [7]

$$f_{\Delta Vt}(\delta V_t) = \frac{1}{\sqrt{2\pi}} \frac{WL}{A_{V_t}^2} \exp\left[-\frac{WL\,\delta V_t^{\,2}}{2A_{V_t}^2} \right] \tag{1}$$

where A_{Vt} , δV_t , W and L denote the proportional constant of random threshold voltage variation, any sampled value of ΔV_t , channel width and channel length respectively [7].

Unlike [7], the other variations apart from ΔV_t are also taken into account. Even though each of these variations is small, their overall combination significantly affects the transistor performance. So, such combination is considered. The variation in the current factor ($\Delta\beta$) is the result of the cited combined variations. Obviously, the current factor (β) can be given by [5]

$$\beta = \mu C_{ox} \frac{W}{L} \tag{2}$$

where μ and C_{ox} denote the carrier mobility and the gate oxide capacitance of the MOS transistor respectively.

It can be seen that the variations in μ, C_{ox}, W and L and yield $\Delta\beta$ because β is the function of μ, C_{ox}, W and L as shown above. This can be quantitatively stated that since there is none of any correlation between μ, C_{ox}, W and L, the variance of $\Delta\beta$ ($\sigma^2(\Delta\beta)$) can be given by [1]

$$\sigma^2(\Delta\beta) = \sigma^2(\Delta\mu) + \sigma^2(\Delta C_{ox}) + \sigma^2(\Delta W) + \sigma^2(\Delta L) \tag{3}$$

where $\sigma^2(\Delta\mu)$, $\sigma^2(\Delta C_{ox})$, $\sigma^2(\Delta W)$ and $\sigma^2(\Delta L)$ denotes the variances of $\Delta\mu$, ΔC_{ox}, ΔW and ΔL which are the variations in μ, C_{ox}, W and L respectively. It can be seen from (3) that $\sigma^2(\Delta\beta)$ is the combined results of $\sigma^2(\Delta\mu)$, $\sigma^2(\Delta C_{ox})$, $\sigma^2(\Delta W)$ and $\sigma^2(\Delta L)$.

For convenience, the per-unit form of $\Delta\beta$ denoted by $\Delta\beta/\beta$ is always preferable. Such $\Delta\beta/\beta$ is adopted in many recent researches for example [9], [10] and [11]. So, this research has also been conducted by following this fashion. Furthermore, since the often cited Pelgrom's model for $\Delta\beta/\beta$ [1] has been applied in recent researches such as [10] etc., it is also adopted here as well. As the devices are closely spaced in the nanoscale regime, the distribution function of $\Delta\beta/\beta$ can be given by

$$f_{\frac{\Delta\beta}{\beta}}\left(\frac{\delta\beta}{\beta}\right) = \frac{1}{\sqrt{2\pi}} \frac{WL}{A_\beta^2} \exp\left[-\frac{WL\left(\frac{\delta\beta}{\beta}\right)^2}{2A_\beta^2} \right] \tag{4}$$

where A_β and $\delta\beta/\beta$ denote the proportional constant of $\Delta\beta/\beta$ and any sampled value of $\Delta\beta/\beta$ respectively [1, 5 and 10]. In the upcoming subsection, parts of the proposed models for the triode region of operation will be discussed.

A. Triode region models

At the circuit level, the key parameter for large signal operations is I_d as mentioned above. So, the modelling for large signal is focused on the random variation in the drain current (ΔI_d). Hence, the resulting large signal models are the analytical expression of ΔI_d along with its distribution function which will be now presented by starting from their derivations.

In the ideal situation where any random variation can be neglected, I_d of the nanoscale transistor operated in the triode region of operation can be given in term of β by [8]

$$I_{d(ideal)} = \beta \left[\frac{2(V_{gs} - V_t)V_{ds} - V_{ds}^2}{1 + \frac{V_{ds}}{V_c}} \right] \tag{5}$$

where V_{gs}, V_{ds}, V_t and V_c denote gate-source voltage, drain-source voltage, threshold voltage and critical voltage respectively. It can be seen that the above $I_{d(ideal)}$ is actually similar to that proposed in [7] by using the following simple relationship

$$V_c = \frac{Lv_{sat}}{\mu} \tag{6}$$

where v_{sat} denotes the saturation velocity. However $I_{d(ideal)}$ as in (5) is preferable here due to its simplicity. Obviously, $I_{d(ideal)}$ is a deterministic variable.

Including the effect of ΔV_t and $\Delta\beta$ which are random variables as stated earlier, I_d becomes a random variable and can be given based on $\Delta\beta/\beta$ by

$$I_d(\Delta V_t, \Delta\beta/\beta) = \beta(1 + (\Delta\beta/\beta)) \left[\frac{2(V_{gs} - (V_t + \Delta V_t))V_{ds} - V_{ds}^2}{1 + \frac{V_{ds}}{V_c}} \right] \tag{7}$$

So, the resulting ΔI_d can be simply determined from (6) and (7) as a linear function of ΔV_t and $\Delta\beta/\beta$ by using the fact that $V_t >> \Delta V_t$ as follows

$$\Delta I_d = \beta\left[(\Delta\beta/\beta)(2(V_{gs} - V_t) - V_{ds} - 2\Delta V_t)\right]\left[\frac{1}{V_{ds}} + \frac{1}{V_c}\right]^{-1} \tag{8}$$

Obviously, ΔI_d is a random variable and its distribution function can be simply given by (9). At this point, it can be stated that parts of the proposed models for the large signal parameter of the transistor operates in the triode region are given by (8) and (9). This can be alternatively stated that ΔI_d is normally distributed with mean ($\overline{\Delta I_d}$) and variance ($\sigma_{\Delta I_d}^2$) given by (10) and (11) respectively.

$$f_{\Delta I_d}(\delta I_d)=\frac{1}{\sqrt{2\pi}\left\{\left[\frac{\beta A_\beta(2(V_{gs}-V_t)-V_{ds})}{\sqrt{WL}\left[\frac{1}{V_{ds}}+\frac{1}{V_c}\right]}\right]^2+\left[\frac{2A_{V_t}\beta}{\sqrt{WL}\left[\frac{1}{V_{ds}}+\frac{1}{V_c}\right]}\right]^2\right\}^{\frac{1}{2}}}$$
$$\times\exp\left\{-\frac{\delta I_d^2}{2\left\{\left[\frac{\beta A_\beta(2(V_{gs}-V_t)-V_{ds})}{\sqrt{WL}\left[\frac{1}{V_{ds}}+\frac{1}{V_c}\right]}\right]^2+\left[\frac{2A_{V_t}\beta}{\sqrt{WL}\left[\frac{1}{V_{ds}}+\frac{1}{V_c}\right]}\right]^2\right\}}\right\} \quad (9)$$

$$\overline{\Delta I_d}=\int_{-\infty}^{\infty}\delta I_d f_{\Delta I_d}(\delta I_d)d\delta I_d=0 \quad (10)$$

$$\sigma^2_{\Delta I_d}=\int_{-\infty}^{\infty}(\delta I_d-\overline{\Delta I_d})^2 f_{\Delta I_d}(\delta I_d)d\delta I_d$$
$$=\left[\frac{\beta A_\beta(2(V_{gs}-V_t)-V_{ds})}{\sqrt{WL}\left[\frac{1}{V_{ds}}+\frac{1}{V_c}\right]}\right]^2+\left[\frac{2A_{V_t}\beta}{\sqrt{WL}\left[\frac{1}{V_{ds}}+\frac{1}{V_c}\right]}\right]^2 \quad (11)$$

On the other hand, the key parameter for small signal triode region operation is the drain-source conductance (g_{ds}) since the transistor behaves as an active resistor in this region. So, the modelling for small signal parameter of the transistor in the triode region is focused on the random variation in the drain-source conductance (Δg_{ds}). Hence, the resulting small signal models in this case are the analytical expression of Δg_{ds} and its distribution function which will be now presented by also starting from their derivations.

In the ideal situation, g_{ds} of the nanoscale transistor operated in the triode region of operation can be given by using (5) as

$$g_{ds(ideal)}=\frac{\beta V_c(2V_t-2V_{ds}V_c-2V_{gs}-V_{ds})^2}{(V_{ds}+V_c)^2} \quad (12)$$

Obviously, $g_{ds\,(ideal)}$ is a deterministic variable. Including the effect of ΔV_t and $\Delta\beta$, g_{ds} which is a random variable and can be given based on ΔV_t and $\Delta\beta/\beta$ by

$$g_{ds}(\Delta V_t,\Delta\beta/\beta)=[\beta(1+(\Delta\beta/\beta))V_c(2(V_t+\Delta V_t)$$
$$-2V_{ds}V_c-2V_{gs}-V_{ds})^2](V_{ds}+V_c)^{-2} \quad (13)$$

So, the resulting Δg_{ds} can be simply determined from (8) and (9) as a linear function of ΔV_t and $\Delta\beta/\beta$ by using the fact that $\beta >>\Delta\beta$ as follows

$$\Delta g_{ds}=\frac{\beta(2V_t-2V_{ds}V_c-2V_{gs}-V_{ds}^2)V_c(\Delta\beta/\beta)}{(V_{ds}+V_c)^2}+\frac{2\beta V_c\Delta V_t}{(V_{ds}+V_c)^2} \quad (14)$$

Obviously, Δg_{ds} is a random variable where its distribution function can be simply given by

$$f_{\Delta g_{ds}}(\delta g_{ds})=\frac{1}{\sqrt{2\pi}}\left\{\left[\frac{\beta(2V_t-2V_{ds}V_c-2V_{gs}-V_{ds}^2)V_cA_\beta}{\sqrt{WL}(V_{ds}+V_c)^2}\right]^2+\left[\frac{2\beta V_cA_{V_t}}{\sqrt{WL}(V_{ds}+V_c)^2}\right]^2\right\}^{-\frac{1}{2}}$$
$$\times\exp\left\{-\frac{1}{2}\left\{\left[\frac{\beta(2V_t-2V_{ds}V_c-2V_{gs}-V_{ds}^2)V_cA_\beta}{\sqrt{WL}(V_{ds}+V_c)^2}\right]^2+\left[\frac{2\beta V_cA_{V_t}}{\sqrt{WL}(V_{ds}+V_c)^2}\right]^2\right\}^{-1}\delta g_{ds}^2\right\}$$
$$(15)$$

At this point, it can be stated that parts of the proposed models for the small signal parameter of the transistor operates in the triode region are given by (14) and (15). This can be alternatively stated that Δg_{ds} is normally distributed with the following mean ($\overline{\Delta g_{ds}}$) and variance ($\sigma^2_{\Delta g_{ds}}$).

$$\overline{\Delta g_{ds}}=\int_{-\infty}^{\infty}\delta g_{ds}f_{\Delta g_{ds}}(\delta g_{ds})d\delta g_{ds}=0 \quad (16)$$

$$\sigma^2_{\Delta g_{ds}}=\int_{-\infty}^{\infty}(\delta g_{ds}-\overline{\Delta g_{ds}})^2 f_{\Delta g_{ds}}(\delta g_{ds})d\delta g_{ds}$$
$$=\left[\frac{\beta(2V_t-2V_{ds}V_c-2V_{gs}-V_{ds}^2)V_cA_\beta}{\sqrt{WL}(V_{ds}+V_c)^2}\right]^2+\left[\frac{2\beta V_cA_{V_t}}{\sqrt{WL}(V_{ds}+V_c)^2}\right]^2 \quad (17)$$

It can be seen from (8), (9), (14) and (15) as parts of the proposed models that both ΔI_d and Δg_{ds} which are the variations at the circuit level, are now analytically expressed in terms of their physical level causes. The physical level causes and their relationships to the resulting circuit level variations are now revealed in both deterministic and probabilistic senses. As expected, elimination of the gap between such two levels is now accomplished. In the next subsection, the parts of the proposed model for the saturation region of operation will be introduced.

B. Saturation region model

Similarly to the triode region of operation, the key large signal parameter is I_d. So, the modelling for large signal in this case is also focused on ΔI_d and the resulting models are the analytical expression of ΔI_d along with its distribution function as well.

Ideally, I_d of the nanoscale transistor operated in the saturation region can be given in term of β by [8]

$$I_{d(ideal)}=\beta(V_{gs}-V_t)V_c \quad (18)$$

Obviously, $I_{d(ideal)}$ is a deterministic variable and is actually similar to that proposed in [7] by using (6).

Including the effect of ΔV_t and $\Delta\beta$, I_d becomes a random variable given by

$$I_d(\Delta V_t, \Delta\beta/\beta) = \beta(1+(\Delta\beta/\beta))(V_{gs}-(V_t+\Delta V_t))V_c \quad (19)$$

So, the resulting ΔI_d can be simply determined from (18) and (19) as a linear function of ΔV_t and $\Delta\beta/\beta$ by using the fact that $\beta >> \Delta\beta$ as follows

$$\Delta I_d = \beta\left[((\Delta\beta/\beta)(V_{gs}-V_t)) - \Delta V_t)\right]V_c \quad (20)$$

Obviously, ΔI_d is a random variable and its distribution function can be simply given by

$$f_{\Delta I_d}(\delta I_d) = \frac{1}{\sqrt{2\pi}}\left\{\frac{1}{WL}\left\{\left[\beta A_\beta (V_{gs}-V_t)V_c\right]^2 - \left[\beta A_{V_t} V_c\right]^2\right\}\right\}^{-\frac{1}{2}} \times \exp\left\{-\frac{1}{2}\left\{\frac{1}{WL}\left\{\left[\beta A_\beta (V_{gs}-V_t)V_c\right]^2 - \left[\beta A_{V_t} V_c\right]^2\right\}\right\}^{-1}\delta I_d^2\right\} \quad (21)$$

At this point, it can be stated that parts of the proposed models for the large signal parameter of the transistor operates in the saturation region are given by (20) and (21). This can be alternatively stated that ΔI_d is normally distributed with the following mean ($\overline{\Delta I_d}$) and variance ($\sigma^2_{\Delta I_d}$) given by.

$$\overline{\Delta I_d} = \int_{-\infty}^{\infty} \delta I_d f_{\Delta I_d}(\delta I_d) d\delta I_d = 0 \quad (22)$$

$$\sigma^2_{\Delta I_d} = \int_{-\infty}^{\infty} (\delta I_d - \overline{\Delta I_d})^2 f_{\Delta I_d}(\delta I_d) d\delta I_d = \frac{1}{WL}\left\{\left[\beta A_\beta (V_{gs}-V_t)V_c\right]^2 - \left[\beta A_{V_t} V_c\right]^2\right\} \quad (23)$$

On the other hand, the key small signal parameter for the operation in the saturation region is the transconductance (g_m) since the transistor behaves as a transconductor. So, the modelling in this case is focused on the random variation in the transconductance (Δg_m). Hence, the resulting small signal models are the analytical expression of Δg_m and its distribution function. In the ideal situation, g_m of the nanoscale transistor can be given by using (18) as

$$g_{m(ideal)} = \beta V_c \quad (24)$$

Obviously, $g_{m(ideal)}$ is a deterministic variable. Including the effect of ΔV_t and $\Delta\beta$, g_m can be given as a function of ΔV_t and $\Delta\beta/\beta$ by

$$g_m(\Delta V_t, \Delta\beta/\beta) = \beta(1+(\Delta\beta/\beta))V_c \quad (25)$$

So, the resulting Δg_m can be simply determined from (24) and (25) as

$$\Delta g_m = \beta V_c(\Delta\beta/\beta) \quad (26)$$

Of course, Δg_m is a random variable where its distribution function can be simply given by

$$f_{\Delta g_m}(\delta g_m) = \frac{1}{\sqrt{2\pi}}\left\{\frac{(\beta A_\beta V_c)^2}{WL}\right\}^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}\left\{\frac{(\beta A_\beta V_c)^2}{WL}\right\}^{-1}\delta g_m^2\right\} \quad (27)$$

At this point, it can be stated that parts of the proposed models for the small signal parameter of the transistor operates in the saturation region are given by (26) and (27). This can be alternatively stated that Δg_m is normally distributed with the following mean ($\overline{\Delta g_m}$) and variance ($\sigma^2_{\Delta g_m}$) given by.

$$\overline{\Delta g_m} = \int_{-\infty}^{\infty} \delta g_m f_{\Delta g_m}(\delta g_m) d\delta g_m = 0 \quad (28)$$

$$\sigma^2_{\Delta g_m} = \int_{-\infty}^{\infty} (\delta g_m - \overline{\Delta g_m})^2 f_{\Delta g_m}(\delta g_m) d\delta g_m = \frac{(\beta A_\beta V_c)^2}{WL} \quad (29)$$

Similarly to the triode region case, it can be seen from (20), (21), (26) and (27) as parts of the proposed models that both ΔI_d and Δg_m which are the circuit level variations, are now analytically expressed in terms of their causes at the physical level as well. Hence, the similar revelation and elimination of the design gap are also accomplished.

At this point, it can be observed that both ΔV_t and $\Delta\beta$ affect the ΔI_d of both regions and Δg_{ds} where as only $\Delta\beta$ affects Δg_m. In the other words, without the inclusion of other variations apart from ΔV_t, ΔI_d of both regions along with Δg_{ds} cannot be accurately modelled and Δg_m cannot even be seen although it exists in reality. Hence, inclusion of such other variations has been found to be crucial.

It has been mentioned above that an obvious benefit of the proposed models is the ability to reveal the physical level causes along with their relationships to the resulting variations in the key parameters of any circuits and systems. For the illustration of this point, consider a single transistor amplifier depicted below.

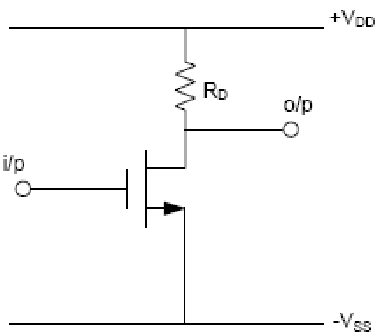

Fig.1. A single transistor amplifier

The most important key parameter for this circuit is the voltage gain (A_v) which can be approximately given by

$$A_v = -g_m R_D \quad (30)$$

Without the proposed models, the variation in A_v must be analysed in the traditional fashion which such variation can be modelled in term of the voltage gain variance ($\sigma^2_{A_v}$) and can be given in term of the transconductance variance (σ^2_{gm}) by

$$\sigma^2_{A_v} = S^{A_v}_{gm}\sigma^2_{gm} = R^2_D\sigma^2_{gm} \quad (31)$$

where $S^{A_v}_{gm}$ denotes the sensitivity of A_v to g_m which can be defined as $S^{A_v}_{gm} = \frac{\partial A_v}{\partial g_m}$ . .

It can be seen that only the relationship between the variations in A_v and g_m which are both at the circuit level can be observed and the conclusion that the variation in g_m is the cause of that in A_v is drawn. Unfortunately, the physical level causes of such variation in A_v which is the primary causes and the relationship between such causes and such variation are unseen. A design gap between the physical level and circuit one existed.

On the other hand, if the proposed models have been applied to this analysis, A_v can be given by

$$A_v = -g_{m(ideal)}R_D - \Delta g_m R_D \quad (32)$$

According to the definition of $g_{m(ideal)}$, the ideal A_v is given by $A_{v(ideal)} = -g_{m(ideal)}R_D$. So, the variation in A_v (ΔA_v) can be simply found as

$$\Delta A_v = -\Delta g_m R_D \quad (33)$$

With parts of the proposed models for Δg_m, ΔA_v and its distribution function can be formulated as

$$\Delta A_v = -\beta V_c R_D(\Delta\beta/\beta) \quad (34)$$

$$f_{\Delta A_v}(\delta A_v) = \frac{1}{\sqrt{2\pi}}\left\{\frac{(\beta A_\beta R_D V_c)^2}{WL}\right\}^{-\frac{1}{2}} \times \exp\left\{-\frac{1}{2}\left\{\frac{(\beta A_\beta R_D V_c)^2}{WL}\right\}^{-1}\delta A_v^2\right\} \quad (35)$$

In the other words, ΔA_v is normally distributed with the following mean ($\overline{\Delta A_v}$) and variance ($\sigma^2_{\Delta A_v}$).

$$\overline{\Delta A_v} = \int_{-\infty}^{\infty}\delta A_v f_{\Delta A_v}(\delta A_v)d\delta A_v = 0 \quad (36)$$

$$\sigma^2_{\Delta A_v} = \int_{-\infty}^{\infty}(\delta A_v - \overline{\Delta A_v})^2 f_{\Delta A_v}(\delta A_v)d\delta A_v = \frac{(\beta A_\beta V_c R_D)^2}{WL} = \frac{(\mu C_{ox}A_\beta V_c\, R_D)^2 W}{L^3} \quad (37)$$

It can be seen from (34) and (35) that physical level causes and the relationship between such causes and ΔA_v have been revealed in both deterministic and probabilistic senses. According to (2) and (3), these causes are $\Delta\mu$, ΔC_{ox}, ΔW and ΔL. Obviously, ΔA_v is proportional to $\Delta\mu$, ΔC_{ox} and ΔW and inversely proportional to ΔL. So, a design guideline can be stated at this point that $\Delta\mu$, ΔC_{ox} and ΔW must be minimized in order to reduce ΔA_v. Furthermore, the important statistical parameters of ΔA_v can be analytically derived as shown in (36) and (37) which another design guideline that for any certain level of technology W and R_D must be minimized in order to reduce $\sigma^2_{\Delta A_v}$ is yielded. These achievements cannot be obtained without the usage of the proposed models.

III. The Verifications

The verifications of the proposed models have been performed in both qualitative and quantitative aspects. In the qualitative sense, the estimated distributions of circuit level parameters obtained from the models have been graphically compared to their counterparts obtained from the Monte Carlo SPICE simulations of the benchmark circuits at 65 nm process technology. On the other hand for the quantitative point of view, numbers of Kolmogorov-Smirnof goodness of fit test (KS-test) have been performed by using the data obtained from the qualitative verifications.

Since the KS-test relies on the cumulative distribution function, it is worthy to derive the cumulative distribution function forms of (9), (15), (21) and (27) at this point. Such cumulative distribution function forms of (9) and (15) which are belonged to the triode region can be given by

$$F_{\Delta I_d}(\delta I_d) = \int_{-\infty}^{\delta I_d} f_{\Delta I_d}(u)du = \frac{1}{2}\left\{1 + erf\left[\frac{1}{\sqrt{2}}\left\{\left[\frac{\beta A_\beta(2(V_{gs}-V_t)-V_{ds})}{\sqrt{WL}\left[\frac{1}{V_{ds}}+\frac{1}{V_c}\right]}\right]^2 + \left[\frac{2A_{V_t}\beta}{\sqrt{WL}\left[\frac{1}{V_{ds}}+\frac{1}{V_c}\right]}\right]^2\right\}^{-\frac{1}{2}}\delta I_d\right]\right\} \quad (38)$$

$$F_{\Delta g_{ds}}(\delta g_{ds}) = \int_{-\infty}^{\delta g_{ds}} f_{\Delta g_{ds}}(u)du$$
$$\frac{1}{2}\left\{1 + erf\left[\frac{\delta g_{ds}}{\sqrt{2}\left\{\left[\frac{\beta(2V_t - 2V_{ds}V_c - 2V_{gs} - V_{ds}^2)V_c A_\beta}{\sqrt{WL}(V_{ds}+V_c)^2}\right]^2 + \left[\frac{2\beta V_c A_{V_t}}{\sqrt{WL}(V_{ds}+V_c)^2}\right]^2\right\}^{\frac{1}{2}}}\right]\right\} \quad (39)$$

On the other hand, those of (21) and (27) which are of the saturation region can be given by

$$F_{\Delta I_d}(\delta I_d) = \int_{-\infty}^{\delta I_d} f_{\Delta I_d}(u)du = \frac{1}{2}\left\{1 + erf\left[\frac{1}{\sqrt{2}}\left\{\frac{1}{WL}\left\{[\beta A_\beta(V_{gs}-V_t)V_c]^2 - [\beta A_{V_t}V_c]^2\right\}\right\}^{-\frac{1}{2}}\delta I_d\right]\right\} \quad (40)$$

$$F_{\Delta g_m}(\delta g_m) = \int_{-\infty}^{\delta g_m} f_{\Delta g_m}(u)du = \frac{1}{2}\left\{1 + erf\left[\left(\frac{\sqrt{WL/2}}{\beta A_\beta V_c}\right)\delta g_m\right]\right\} \quad (41)$$

where $erf(x)$ denotes the error function of any arbitrary variable, x which can be mathematically defined as $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x \exp(-u^2)du$.

According to [12, 13], the strategy of the KS-test is to performed the comparison of the K-S test statistic (KS) and the critical value (c) where it can be stated that any model fits its target data set if and only if its KS is not exceed its c [12, 13]. For this research, KS can be defined as

$$KS = \max_{\delta x}\{\left|F_{\Delta x}(\delta x)\big|_{circuit}\right| - \left|F_{\Delta x}(\delta x)\big|_{model}\right|\} \quad (42)$$

where x denotes any circuit level parameters under consideration which can be either I_d , g_m or g_{ds} while $F_{\Delta x}(\delta x)\big|_{model}$ and $F_{\Delta x}(\delta x)\big|_{circuit}$ represent the estimated distribution of any x obtained from the proposed models and their counterparts obtained from the test circuits respectively. On the other hand, as the confidence level of the test is 99% or $\alpha = 0.01$ in the other words, c can be given by [13]

$$c = \frac{1.63}{\sqrt{n}} \quad (43)$$

where n denotes the number of samples.

Before proceed further, it should be mentioned here that the verifications of the proposed models have been performed by using n = 100 which yields c = 0.163. In the upcoming subsection, the verification of the models for the triode region will be discussed.

A. Triode region models verification

Similarly to [7], a single transistor active resistor has also been adopted as the benchmark circuit. This circuit is depicted in Fig.2. As the qualitative verification, the graphical comparisons for the distributions of per-unit change in I_d and g_{ds} denoted by $\Delta I_d/I_d$ and $\Delta g_{ds}/g_{ds}$ respectively are depicted in Fig. 3 and Fig.4 which strong agreements between the estimates and their counterparts can be observed. Hence, proposed triode region models have been qualitatively verified as highly accurate.

For the quantitative verification, it can be seen by using (42) that the resulting KS for x = I_d can be found as KS = 0.15127 and that for x = g_{ds} is KS = 0.08524 which are both smaller than c = 0.163. This means that the proposed triode region models can fit the variations in I_d and g_{ds} obtained from the test circuit with 99% confidence. At this point, the triode region models are both qualitatively and quantitatively verified as highly accurate.

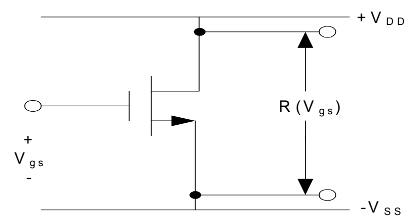

Fig.2: A single MOS active resistor

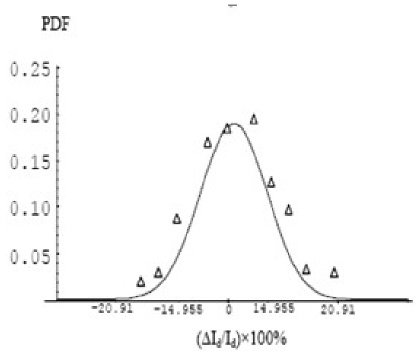

Fig.3. Triode region distribution comparison for $\Delta I_d/I_d$: estimated distribution from the model (line), actual distribution from the test circuit (Δ).

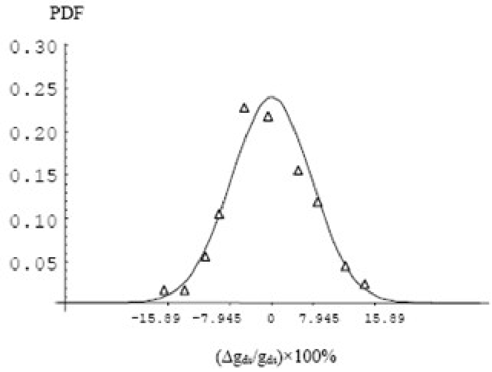

Fig.4: Triode region distribution comparison for $\Delta g_{ds}/g_{ds}$: estimated distribution from the model (line), actual distribution from the test circuit (Δ).

B. Saturation region models verification

On the other hand, for the case of the saturation region models, a diode connected transistor has been chosen as the benchmark circuit for the variation in drain current similarly to the saturation region verification proposed in [7] where as a single transistor amplifier depicted in Fig.1 has been chosen for the variation in transconductance. Such diode connected transistor can be depicted in Fig.5. Similarly to the qualitative verification of the triode region models, the graphical comparisons of the Δx/x distributions are performed as depicted in Fig. 6 and Fig.7 for x = I_d and x = g_m respectively. In this case, strong agreements between the estimates and their counterparts can also be observed. At this point, the proposed saturation region models have been qualitatively verified as highly accurate.

For the verifications in the quantitative aspect, by (42), the resulting KS for x = I_d can be found as KS = 0.08943 and that for x = g_m is KS = 0.14872 which are both smaller than c = 0.163. This means that the proposed saturation region models can also fit the variations in I_d and g_m obtained from their corresponding test circuits with 99% confidence. Here, the saturation region models have also been both qualitatively and quantitatively verified as highly accurate.

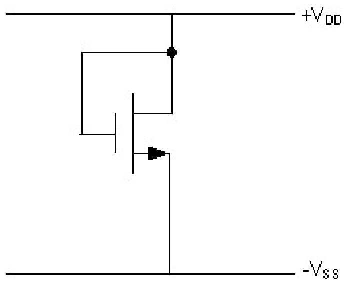

Fig.5: A diode connected transistor

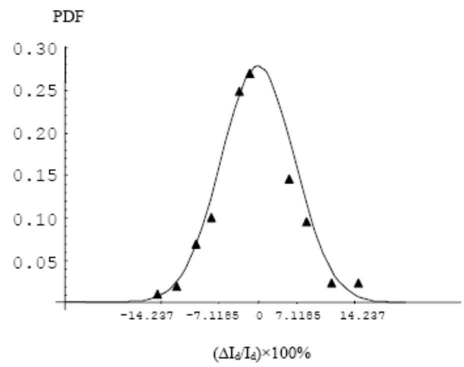

Fig.6: Saturation region distribution comparison for $\Delta I_d/I_d$: estimated PDF from the model (line), actual PDF from the test circuit (▲)

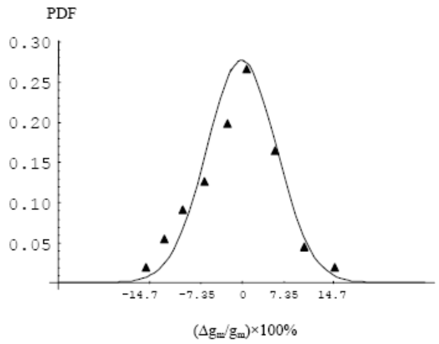

Fig.7: Saturation region distribution comparison for $\Delta g_m/g_m$: estimated PDF from the model (line), actual PDF from the test circuit (▲)

IV. Conclusions

The complete analytical deterministic/probabilistic models of random variations in circuit level parameters for any nanoscale MOS transistor, for both triode and saturation regions have been proposed by using the up to dated nanoscale regime MOS equations [8] as the foundations. The proposed models have been verified at the nanoscale regime by using the Monte Carlo SPICE simulations and the KS-tests. The chosen benchmark circuits are a single transistor active resistor for the triode region models along with a diode connected transistor and a single transistor amplifier for the saturation region models. These models are very accurate since they can fit the random variations in circuit level parameters obtained from the test circuits with 99% confidence. Furthermore, they can effectively eliminate the gap between the circuit level and physical level design and also expected to be the potential mathematical foundations for implementating the ECAD cell libraries for nanoscale MOS transistors. So, the proposed models are obviously efficient for the statistical/variability aware design of various nanoscale CMOS analog/mixed signal circuits and systems.

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Rawid Banchuin, biography and photograph not available at the time of publishing.