Daily Discharge Forecasting Using Support Vector Machine

Mahdi Moharrampour, Abdulhamid Mehrabi, and Mahya Katouzi

Abstract—Support Vector Machine (SVM) is a kind of learning machine for simulation or prediction. In this paper, Support Vector Machine (SVM) is used to forecast daily river flow and the results of these models are compared with observed daily values. Daily river flow data on Ghara-soo river in north of Iran are used in this study. The daily flow and rain data of station on Ghara-soo as exit discharge and three station of this Catchment Names: shast kalate, ziyarat and Kurd kuy are used to train and test the developed models. The observed data that are used in this study start from 1992 to 2010 in 18 year’s period (6550 days). 75% of the whole data set are used for training the models and 25% of the whole data set are used for testing step. In this regard, five kind of different input data that affect the river flow has been identified and based on this method, the river flow is predicted. In this research, predicted data are compared with actual data through the RMSE index. More than 4 rain measurement stations are existed over this river, but because of lack of statistics for all stations, in this research 4 stations are used. Gharasoo station as exit discharge of this basin and Ziarat, Shastkalate and Kordkooy as input of this basin in three different locations are used. Table I. [3], [4]

TABLE I: SPECIFICATION OF GHARASOO BASIN STATIONS

<table>
<thead>
<tr>
<th>Province</th>
<th>Location</th>
<th>River</th>
<th>Longitude</th>
<th>Latitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Golestian</td>
<td>Gharasoo</td>
<td>Gharasoo</td>
<td>54-03-00</td>
<td>36-50-00</td>
</tr>
<tr>
<td>Golestian</td>
<td>Naharkhoran</td>
<td>Ziarat</td>
<td>54-28-00</td>
<td>36-46-00</td>
</tr>
<tr>
<td>Golestian</td>
<td>Shastkalate</td>
<td>Shastkalate</td>
<td>54-20-00</td>
<td>36-45-00</td>
</tr>
<tr>
<td>Golestian</td>
<td>pole jadde</td>
<td>Kordkooy</td>
<td>54-05-00</td>
<td>36-47-00</td>
</tr>
</tbody>
</table>

More than 4 rain measurement stations are existed over this river, but because of lack of statistics for all stations, in this research 4 stations are used. Gharaso station as exit discharge of this basin and Ziarat, Shastkalate and Kordkooy as input of this basin in three different locations are used.

III. PREPROCESSING DATA

Preprocessing of data includes selection of effective variable, selection of training and test patterns and normalizing the patterns. The goal of normalizing is that all values in one pattern are in a range. Pattern normalizing exchanges all values to a specified interval such as [0-1] or [-1,1]. After normalizing all patterns, period of case study was selected between 1989 to 2007 (18 years). For this period, there are 6550 daily patterns for every station. 75% of these data are used for support vector machine training and 25% of these data are used for test. Fig. 2 shows daily flow hydrograph of Gharasoo River for training period and Fig. 3 shows daily flow hydrograph of Gharasoo River for test period.[3],[4]
Support Vector Machines is based on statistical learning theory. According to the Structural Risk Minimization (SRM) principle, the generalization ability of learning machines depends more on capacity concepts than merely the dimensionality of the space or the number of free parameters of the loss function. Thus, for a given set of observations \((x_1, y_1), \ldots, (x_n, y_n)\), the SRM principle chooses the function \(f_{b*}\) in the subset, for which the guaranteed risk bound, as given by Eq. (1) below, is minimal. In other words, the actual risk is controlled by the two terms given in Eq. (1):

\[
R(a) \leq \text{Remph}(a) + \Omega (n/h) \quad (1)
\]

where the first term is an estimate of the risk and the second term is the confidence interval for this estimate. The parameter \(h\) is called the VC dimension (named after Vapnik and Chervonenkis) of a set of functions. It can be seen as the measure of the capability of a set of functions implementable by the learning machine to best approximate the problem.

SVM is an approximate implementation of the SRM principle. The final approximating function used in SVM for regression is of the form

\[
f(x) = \sum_{i=1}^{n} (a_i - a_i^*) \langle x_i, x \rangle + b
\]

where \(\langle x_i, x \rangle = <\Phi(x_i), \Phi(x)\rangle\) is called the kernel function, which performs the inner product in feature space. \(\Phi(x_i), a_i\) and \(a_i^*\) are Lagrange multipliers. To act as a kernel, a function needs to satisfy Mercer’s condition. The kernel representation offers a powerful alternative for using linear machines in hypothesizing complex real world problems as opposed to Artificial Neural Network based learning paradigms, which use multiple layers of threshold linear functions.

The approximating function is designed to have the smallest \(\varepsilon\) deviation (given as Vapnik’s \(\varepsilon\)-insensitive loss function) from measured targets, \(Y_i\), for all training data. Slack variables, \(\xi_i\) and \(\xi_j^*\), are introduced to account for outliers in the training data. The algorithm computes the value of Lagrange multipliers, \(ai\) and \(aj^*\) by minimizing the following objective function:

Minimize

\[
\sum_{i=1}^{n} a_i - a_i^* = 0
\]

Subject to

\[
a_i - a_i^* \in 0, c
\]

This equation is expressed in the dual form, are given as

Maximize

\[
-\frac{1}{2} \sum_{i,j=1}^{n} a_i - a_i^* \langle x_i, x_j \rangle - \varepsilon \sum_{i=1}^{n} a_i + a_i^* + \sum_{i=1}^{n} y_i \ a_i - a_i^*
\]

Subject to

\[
\sum_{i=1}^{n} a_i - a_i^* = 0
\]

\[
a_i - a_i^* \in 0, c
\]

where \(C\) is a user specified constant and it determines the trade-off between the flatness of \(f(x)\) and the amount of deviation that can be tolerated. The value ‘\(a\)’ refers to the weight factor for obtaining the flattest decision function. It should be noted that the training patterns, appearing in both objective functions of Eq. (4) and in the approximating function of Eq. (2), are in the form of dot products.

It can be shown that all the training patterns within the \(\varepsilon\)-insensitive zone yield \(ai\) and \(aj^*\) as zeros. The remaining non-zero coefficients essentially define the final decision function. The training examples corresponding to these non-vanishing coefficients are called Support Vectors.

Optimal values of \(\varepsilon\), \(C\) and the kernel-specific parameters are to be used for the finalregression estimation. Currently, identification of optimal values for these parameters is mainly conducted on a trial and error process.

As well as the \(\varepsilon\)-insensitive loss function, a quadratic loss function \((\varepsilon = 0)\) may also be used. In this study, the quadratic loss function is preferred over the \(\varepsilon\)-insensitive loss function as the former is less computer memory intensive [1], [2].

V. DESIGN AND PRODUCE THE SIMULATION MODEL BY SUPPORT VECTOR MACHINE

Selection of number and type of model input parameters is so important for SVM training. Since, there is no constant
Accordingly, five below patterns are investigated:

1) \( Q(t) = f \{ P_s(t), P_r(t), P_{sh}(t), P_p(t), P_{sh}(t-1), P_p(t-1), \}

\( P_s(t-1), Q_s(t-1), Q_p(t-1), Q(t-1), Q(t-2) \)\)

2) \( Q(t) = f \{ Q_s(t), Q_p(t-1), Q(t-1), Q(t-2) \} \)

3) \( Q(t) = f \{ P_s(t), P_r(t), P_{sh}(t), P_p(t), P_p(t-1), P_{sh}(t-1), P_p(t-1), Q(t-1), Q(t-2) \} \)

4) \( Q(t) = f \{ Q(t-1), Q(t-2) \} \)

5) \( Q(t) = f \{ P_s(t), P_r(t), P_{sh}(t), P_p(t), P_p(t-1), P_{sh}(t-1), P_p(t-1), P_s(t-1) \} \)

In these equations:

- \( Q \) : Daily average discharge of Gharasoo station
- \( Q_s \) : Daily average discharge of Naharkhoran station
- \( Q_p \) : Daily average discharge of Polejadde station
- \( P_s \) : Daily average rainfall of Shastkalateh station
- \( P_{sh} \) : Daily average rainfall of Shastkalateh station
- \( P_p \) : Daily average rainfall of Polejadde station
- \( P_{sh} \) : Daily average rainfall of Shastkalateh station
- \( P_p \) : Daily average rainfall of Polejadde station
- \( P_{sh} \) : Daily average rainfall of Gharasoo station

Of course other patterns were built, but result of everyone is so near to our 5 patterns, so we do not repeat them here again.

RMSE parameter is calculated for performance evaluation of these patterns according to training data. The results are shown in Table II As seen in this table, minimum RMSE is for pattern 1. So, pattern 1 could be the best pattern for river flow forecasting.

### TABLE I: THE ARRANGEMENT OF CHANNELS

<table>
<thead>
<tr>
<th>Input pattern</th>
<th>Pattern 1</th>
<th>Pattern 2</th>
<th>Pattern 3</th>
<th>Pattern 4</th>
<th>Pattern 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>0.03040</td>
<td>0.03234</td>
<td>0.03179</td>
<td>0.03347</td>
<td>0.21833</td>
</tr>
</tbody>
</table>

Table II shows that SVM method has a good result with 14 input values Fig. 4 shows a comparison between model output according to test data and real data. RMSE is about 0.034401 here.

![Fig. 4. Comparison between model output and real data (pattern1).](image)

### VI. SENSITIVITY ANALYSIS FOR SVM MODEL INPUTS

After selecting a SVM pattern, SVM parameters should be selected too. SVM model of this case study has one output and many variable inputs (according to patterns). Kernel function selection depends on training data volume and feature vector dimensions. In other words, one Kernel function shall be selected to have learning ability of input data according to parameters. Four type of Kernel function are used for this paper; linear, polynomial, hyperbolic tangent and Gaussian (RBF) Kernel. Table 3 shows the usual Kernel's equations and table4 shows the results of RMSE for a same input and output configuration for pattern No. one [5].

**TABLE III: USUAL KERNEL'S EQUATIONS**

<table>
<thead>
<tr>
<th>FUNCTION</th>
<th>TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K(x_i, x_j) = x_i^T.x_j )</td>
<td>linear</td>
</tr>
<tr>
<td>( K(x_i, x_j) = (\gamma x_i^T.x_j + r)^d )</td>
<td>Polynomial</td>
</tr>
<tr>
<td>( K(x_i, x_j) = \tanh(\gamma x_i^T.x_j) )</td>
<td>hyperbolic tangent</td>
</tr>
<tr>
<td>( K(x_i, x_j) = e^{-(\gamma</td>
<td>x_i-x_j</td>
</tr>
</tbody>
</table>

### TABLE IV: RESULTS OF RMSE FOR A SAME INPUT AND OUTPUT FOR PATTERN NO.ONE.

<table>
<thead>
<tr>
<th>Type</th>
<th>linear</th>
<th>polynomial</th>
<th>hyperbolic tangent</th>
<th>Gaussian (RBF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Results</td>
<td>0.037776</td>
<td>0.10032</td>
<td>0.034401</td>
<td></td>
</tr>
</tbody>
</table>

As seen in Table II, Gaussian (RBF) Kernel has the best results and this type of function is used for river flow forecasting in this paper.

In SVM modeling with LIBSVM software, the goal is to obtain \( C \) and \( \gamma \). For obtaining the proper \( C \) and \( \gamma \), network searching algorithm is used. For this goal, one of the parameters is supposed as a constant and the other is changed to find the minimum of RMSE for the specified Kernel function. After that the parameters are changed (The second parameter as a constant and the first is changed). So sensitivity of Kernel is measured to both parameters. Table (3) shows a sample for network searching algorithm. For the best result in this paper (pattern one with Gaussian Kernel) \( C \) is obtained 0.001 and \( \gamma \) is obtained 10000.

### VII. CONCLUSION

This paper is shown application of support vector machines (SVMs) in predicting daily flows. These prediction capabilities will complement the existing effort in water resource management of the Gharasoo River Basin. In this study first pattern is the best pattern because of its minimum error. Of course, if minimum error and minimum parameters were important for research, the best result would be obtained from pattern No. 4. The approach considers that river flow forecasting mostly depends on last day flow and flow of two days before in Gharasoo River. SVMs are machine learning methodology that have important features including the fact that requirement on kernel and the nature of the optimization problem results in global optima avoiding the danger converging to a local optima. Moreover, the predictions from
SVM offer special advantages as compared to other machine learning techniques like ANN. Unlike ANN, SVM does not require the architecture. The structural risk minimization principle gives SVM the desirable property to generalize well in the unseen data. The dual representation offers the unique advantage of ease in dealing with the high-dimensional input vectors without loss of both generalization accuracy and computational efficiency. The optimization problem formulated for SVM is always uniquely solvable and, thus, does not suffer from the limitation of ways of regularization as in ANN, which may lead them to local minima. Programming in LIBSVM is too easy and too short, so processing is too much fast. SVMs are not without weakness: kernel selections as well as SVM hyper-parameters (C and $\varepsilon$) are still heuristic.

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>C</th>
<th>MSE</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$\epsilon$</th>
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<tbody>
<tr>
<td>10-4</td>
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<td>.037</td>
<td>.038</td>
<td>.037</td>
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<td>102</td>
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</table>

REFERENCES


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