An Improved Least Mean Kurtosis (LMK) Algorithm for Sparse System Identification

Jin Woo Yoo and PooGyeon Park

***Abstract*—This paper proposes an improved least mean kurtosis (LMK) algorithm based on l_0-norm cost for enhancing the filter performance in a sparse system. The LMK adaptive filtering algorithm uses a kurtosis of an estimated error signal to improve the filter performance when the noise contamination is serious. Due to the influence of l_0-norm cost, the proposed LMK algorithm ensures a fast convergence rate and a small steady-state error in sparse system environment. Simulation results verify that the proposed algorithm improves the filter performance for sparse system identification.**

***Index Terms*—Adaptive filter, least mean kurtosis algorithm, sparse system identification.**

I. Introduction

Adaptive filters have been widely used as a general tool for diverse applications such as echo cancellation, noise cancellation, and channel estimation. The least-mean-square (LMS) algorithm and normalized least-mean-square (NLMS) algorithm are the most well-known adaptive filtering algorithms because of their low computational complexity and ease of implementation [1]-[2]. Although LMS and NLMS have practical advantages, they suffer from the performance degradation when the output noise contamination occurs. Therefore, the least mean-fourth (LMF) algorithm [3] has been proposed to overcome the performance degradation due to the output noise contamination, and it can actually achieve a faster convergence rate than LMS and NLMS when the output noise has periodic or uniformly distributed property. The LMF uses the expectation of fourth order of an estimated error signal. However, if the noise signal is Gaussian distributed, the LMF has no benefit compared to LMS and NLMS. For this reason, the least mean kurtosis (LMK) algorithm [4]-[5] has been introduced to be noise robust in a wide range of output noise signals such as impulsive noises, Uniformly distributed noises, Gaussian distributed noises. The recent issue of LMF and LMK algorithms is related to stability problem because they use the cost functions including the high order of error signal. For maintaining the stability of these algorithms, a small step size is used compared with that of LMS, NLMS.

Regrettably, because the above-mentioned algorithms do not reflect the sparse property of a system, the improvement of the filter performance for sparse system identification is possible. A sparse system is a special system whose impulse response mainly consists of near-zero coefficients and very few large coefficients. Although sparse systems have a particular property, they exist in many applications such as digital TV transmission channel estimation, network echo cancellation or underwater channel estimation. Recently, several algorithms have been proposed to improve the filter performance for identifying a sparse system [6]-[7]. Among them, applying the idea of l_0-norm constraint to the LMS algorithm has been introduced to improve the convergence rate of a sparse system [6].

This letter proposes an improved LMK algorithm based on the l_0-norm cost for sparse system identification. We suggest a novel cost function including the l_0-norm cost to obtain an enhanced version of LMK algorithm for a sparse system. The proposed LMK algorithm was experimented in a sparse system, and its performance was compared with that of conventional LMK [4], conventional LMS [1]-[2], and l_0-norm constraint LMS [6].

II. Background

A. Basic Principle of Adaptive Filter

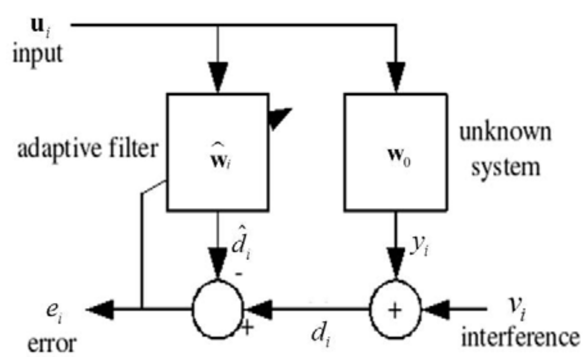

Fig. 1. Block diagram of adaptive filter.

Basically, adaptive filter is to estimate a channel, which means that the goal of adaptive filter finds an accurate $\hat{\mathbf{w}}_i$, the estimate of the ideal filter coefficient $\mathbf{w}_o$. In this paper,

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the proposed LMK algorithm is applied in adaptive filter part to update the filter coefficient $\widehat{\mathbf{w}}_i = [\hat{w}_i(0),...,\hat{w}_i(n-1)]^T$ recursively. $\mathbf{u}_i = [u_i, u_{i-1},...,u_{i-n+1}]^T$ that is an input data goes to adaptive filter and unknown system at the same time. Adaptive filter and unknown system bears the output signals $\hat{d}_i, y_i$, and the measurement noise is added at y_i. The desired system output signal is the sum of y_i and v_i.The difference between $\hat{d}_i$ and d_i is the error signal, which is a feedback factor of adaptive filter. In other words, the filter coefficient $\widehat{\mathbf{w}}_i$ is updated recursively by using the error signal.

B. Sparse System

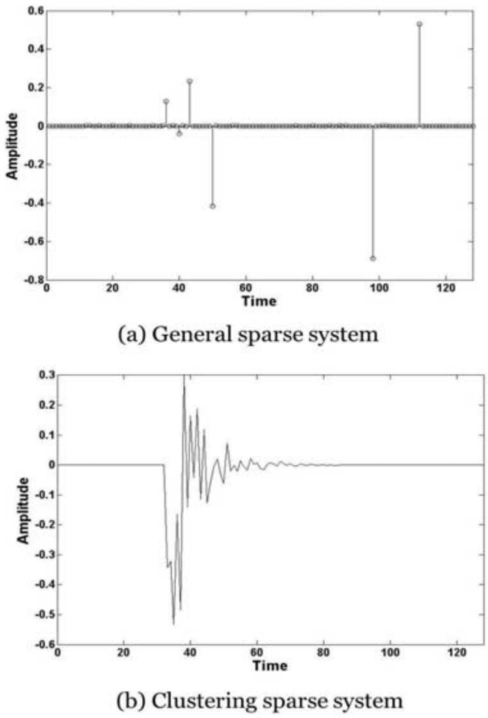

Fig. 2. Two types of sparse system, (a): general sparse system; (b): clustering sparse system.

The impulse response of a sparse system has many near-zero coefficients and very few large coefficients. There are two types of sparse systems, general sparse system and clustering sparse system. As you can see in fig. 2 (a) and (b), unlike general sparse system, clustering sparse system has a gathering of large filter coefficient. In this paper, our target system is general sparse system for identifying the system coefficient.

III. Improved Least Mean Kurtosis Algorithm

A. Conventional Least Mean Kurtosis Algorithm

The cost function of convention LMK algorithm is as follow [4] :

$$J_{LMK}(i) = 3E^2(e_i^2) - E(e_i^4) \tag{1}$$

Through the gradient method, the update equation filter coefficient can be derived as

$$\mathbf{w}_{i+1} = \mathbf{w}_i + \nabla\{J_{LMK}(i)\}$$
$$\mathbf{w}_{i+1} = \mathbf{w}_i + \mu 4(3\sigma_{e_i}^{\ 2} - e_i^2)e_i\mathbf{u}_i \tag{2}$$

where μ is the step size, and $\sigma_{e_i}^{\ 2} = E(e_i^2)$.

Since $\sigma_{e_i}^{\ 2}$ is not reachable value, a smoothing factor is used for estimating $E(e_i^2)$ as like below:

$$\sigma_{e_i}^2 = \alpha\sigma_{e_{i-1}}^2 + e_i^2, \quad 0 < \alpha < 1 \tag{3}$$

B. Proposed LMK Based on l_0-Norm Cost

The proposed cost function of an improved LMK algorithm is obtained by adding the l_0-norm of $\widehat{\mathbf{w}}_i$ at the cost function of the conventional LMK algorithm, as follow [6]:

$$J_p(i) = 3E^2(e_i^2) - E(e_i^4) + \gamma \|\widehat{\mathbf{w}}_i\|_0 \tag{4}$$

where $\|\cdot\|_0$ denotes the l_0-norm that means the number of nonzero entries, and γ is a parameter to adjust the influence of l_0-norm cost.

Through the gradient method, the proposed update equation filter coefficient can be derived as

$$\mathbf{w}_{i+1} = \mathbf{w}_i + \nabla\{J_p(i)\}$$
$$\mathbf{w}_{i+1} = \mathbf{w}_i + \mu[4(3\sigma_{e_i}^{\ 2} - e_i^2)e_i\mathbf{u}_i - \gamma\mathbf{f}(\widehat{\mathbf{w}}_i)] \tag{5}$$

where $\mathbf{f}(\widehat{\mathbf{w}}_i) \triangleq [f(\hat{w}_i(0)),..., f(\hat{w}_i(n-1))]^T$, μ is the step size, and $\sigma_{e_i}^{\ 2} = E(e_i^2)$.

Like the conventional LMK algorithm, because $\sigma_{e_i}^{\ 2}$ is not obtainable value, a smoothing factor is used for estimating $E(e_i^2)$ as like below:

$$\sigma_{e_i}^2 = \alpha\sigma_{e_{i-1}}^2 + e_i^2, \quad 0 < \alpha < 1 \tag{6}$$

C. The description of function $\mathbf{f}(\widehat{\mathbf{w}}_i)$

A widely known approximation of l_0-norm is as follow:

$$\|\widehat{\mathbf{w}}_i\|_0 \approx \sum_{k=0}^{n-1}\left(1 - e^{-\beta|\hat{w}_i(k)|}\right) \tag{7}$$

where the parameter β is a positive integer to determine the range of zero-attraction.

The derivative of (7) with respect to the filter coefficient vector can be expressed component-wisely as

$$f(\widehat{w}_i(k)) = \frac{\partial \|\widehat{\mathbf{w}}_i\|_0}{\partial \widehat{w}_i(k)} = \beta \operatorname{sgn}(\widehat{w}_i(k)) e^{-\beta|\widehat{w}_i(k)|} \qquad \forall 0 \le k < n. \tag{8}$$

For decreasing the computational complexity of (8), the first order Taylor series expansion of the exponential function (9) is adopted by

$$e^{-\beta|x|} \approx \begin{cases} 1-\beta|x|, & |x| \le \dfrac{1}{\beta} \\ 0, & \text{elsewhere.} \end{cases} \tag{9}$$

Moreover, the sign function $\operatorname{sgn}(\cdot)$ is defined as

$$\operatorname{sgn}(x) = \begin{cases} \dfrac{x}{|x|}, & x \ne 0 \\ 0, & \text{elsewhere.} \end{cases} \tag{10}$$

Substituting (9) and (10) into (8), we can present the function $f(\cdot)$ as

$$f(x) = \begin{cases} \beta^2 x + \beta, & -\dfrac{1}{\beta} \le x < 0 \\ \beta^2 x - \beta, & 0 < x \le \dfrac{1}{\beta} \\ 0, & \text{elsewhere.} \end{cases} \tag{11}$$

TABLE I: PSEUDO-CODES OF PROPOSED ALGORITHM

```
Given n, μ , α , β , γ ;
Initial w = zeros(n,1), f = zeros(n,1);

For i=1,2,…
    input u, output d;
    e = d-u'*w;
    f(1:n) = - β *max(0,1- β *abs(w(1:n))).*sign(w(1:n));
    σ e2= α *σ e2+e2;
    w= w+ μ *( 4*(3*σ e2-e2)*e*u + γ *f );
End
```

Table I shows the pseudo-codes of the proposed LMK algorithm.

IV. EXPERIMENTAL RESULTS

We illustrate the performance of the proposed algorithm by performing computer experiments in channel estimation. The channel of the unknown system is generated by a moving average model with 128 taps (n=128). We assume that the adaptive filter and the unknown channel have the same number of taps. Moreover we set 120 near-zero filter coefficients among 128 taps to realize a general sparse system. We also assume that the noise variance, σ_v^2, which is known a priory, because it is easy to be estimated. The input signal $\mathbf{u}_i$ is generated by filtering a white, zero-mean, Gaussian random sequence, which denotes the white input. The signal-to-noise ratio (SNR) is set to 10dB which is defined by $\mathrm{SNR} = 10\log_{10}(E(y_i^2)/E(v_i^2))$ with $y_i = \mathbf{u}_i^T \mathbf{w}_o$. The impulsive noise n_i was generated as $n_i = k_i A_i$, where k_i is a Bernoulli process with a probability of success $P[k_i = 1] = \Pr$, and A_i is zero-mean Gaussian noise with power $\sigma_A^2 = \sigma_y^2$. $\Pr$ was set to 0.001, and it means the probability of occurrence of impulsive noise.

The mean square deviation (MSD) is defined as $\mathrm{MSD} = (\mathbf{w}_o - \widehat{\mathbf{w}}_i)^T(\mathbf{w}_o - \widehat{\mathbf{w}}_i)/\mathbf{w}_o^T\mathbf{w}_o$. The simulation results are obtained by ensemble averaging over 100 independent trials.

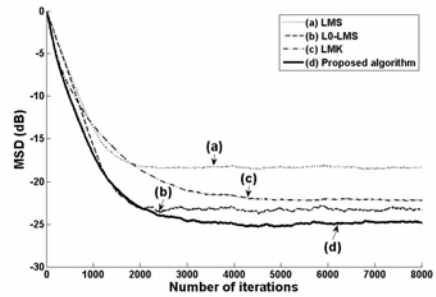

Fig. 3. MSD learning curves for white input, SNR=10 dB

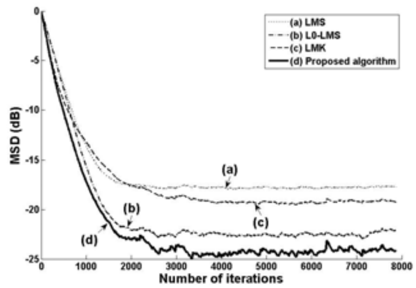

Fig. 4. MSD learning curves for white input when impulsive noises occurs with Pr=0.001.

Fig. 3 shows the MSD learning curves of the conventional LMS algorithm [1]-[2], the l_0-norm constraint LMS algorithm [6], the conventional LMK algorithm [4], and proposed LMK algorithm when the system has white input signals and low SNR. As you can see, the conventional LMK algorithm has a small steady-state error than that of the conventional LMS algorithm due to using high order error signal. On the other hand, the l_0-norm constraint LMS algorithm has faster convergence rate and smaller steady-state error than those of the conventional LMS owing to l_0-norm cost in a sparse system. Similarly, the proposed LMK algorithm has faster convergence rate and smaller steady-state error than those of the convention LMK algorithm in sparse system. Applying l_0-norm concept at LMS algorithm and LMK algorithm, we can find that the l_0-norm cost improves the filter performance in sparse system identification.

Fig. 4 shows the MSD learning curves of the conventional LMS algorithm, the l_0-norm constraint LMS algorithm, the conventional LMK algorithm, and proposed LMK algorithm when the input signal is white with the impulsive noises occurring with $\Pr$ =0.001. Despite the impulsive noises are added at the system output signal, the proposed LMK algorithm attains a fast convergence rate and a small steady-state error compared to the other algorithms.

In fig. 3 and fig 4., the parameters used in simulation are as follow:

the conventional LMS algorithm
(μ =0.002),
the l_0-norm constraint LMS algorithm
(μ =0.002, β =5, κ =0.000008),
the conventional LMK algorithm
(μ =0.0001, α =0.8),
the proposed LMK algorithm
(μ =0.0001, α =0.8, β =5, γ =0.08).

In the last analysis, the proposed LMK algorithm has faster convergence rate and smaller steady-state estimation error than the conventional LMS algorithm, the l_0-norm constraint LMS algorithm, the conventional LMK algorithm when the noise contamination is severe.

V. Conclusion

This paper has proposed an improved LMK algorithm for sparse system identification. Owing to l_0-norm cost, the proposed algorithm is to accelerate the convergence of near-zero coefficients. The experimental results showed that the proposed LMK algorithm accomplished faster convergence rate, smaller steady-state estimation errors, and lower computational complexity than the existing algorithms.

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