Abstract—This paper presents an exact analysis of noise effects of RF oscillators in M-QAM Orthogonal Frequency Division Multiplexing (OFDM) Communication Systems. The phase noise and the amplitude noise effects are studied in the performance of OFDM systems. A theoretical analysis for signal to noise degradation caused by phase and amplitude noise in OFDM bandwidth is presented. It is shown that bit error rate of analytical results is closely match with simulation results for OFDM system using M-QAM modulation.

Index Terms—OFDM, QAM, oscillator noise, signal to noise degradation.

I. INTRODUCTION

Orthogonal frequency division multiplexing is very attractive transmission techniques for future high data rate wireless multimedia communication systems. OFDM has developed for wideband wireless digital communication, used in applications such as Digital Video and Audio Broadcasting (DV&B&DAB), wireless networking and broadband internet access[1]-[3]. The OFDM has advantages such as high spectral efficiency, easy adaptation to severe channel conditions without complex equalization, and efficient implementation using FFT. It yields however some disadvantages, such as the sensitivity to frequency offset, the high peak-to-average-power ratio (PAPR) and the sensitivity to noises [1], [4], [5].

It is very important to exactly predict and analyze the oscillator noise influences in OFDM communication system and quantify the tolerable level of noise. An oscillator is a system that generates a periodic signal with a specified or controllable frequency. There are the two kinds of noise in an oscillator output, namely amplitude noise and phase noise [6]. Phase noise effects in OFDM have been analyzed in many papers by several authors [4],[5],[7]-[19] but they did not consider the effect of amplitude noise and always ignore it relative to phase noise without any acceptable reason.

The purpose of this paper is to extract the theoretical analysis of impact of oscillator phase and amplitude noise on the SNR (hence the BER) performance of MQAM-OFDM signals over AWGN channel and quantify the tolerable level of phase and amplitude noises. This paper is organized as follows: In section II we introduce OFDM system and present a block diagram of the system. In section III we derive the theoretical analysis of phase and amplitude noise effect in OFDM system and introduce power density function of oscillator phase noise. Section IV gives performance analysis and simulation results and conclusions are presented in Section V.

II. OFDM SYSTEM

Orthogonal frequency division multiplexing is a Frequency Division Multiplexing (FDM) scheme utilized as a digital multi-carrier modulation method. A large number of closely-spaced orthogonal sub-carriers are used to carry data. The data are divided into several parallel data streams or channels, one for each sub-carrier. Each sub-carrier is modulated with a conventional modulation scheme such as Quadrature Amplitude Modulation (QAM) or Phase Shift Keying (PSK) at a low symbol rate, maintaining total data rates similar to conventional single-carrier modulation schemes in the same bandwidth [2],[20]. Fig.1 shows a block diagram of OFDM transceiver system. $S(n)$ is a serial stream of binary digits. By inverse multiplexing, these are first demultiplexed into N parallel streams, and each one mapped to a (possibly complex) symbol stream using some modulation constellation (QAM, PSK, etc.). An inverse FFT is computed on each set of symbols, giving a set of complex time-domain samples. These samples are then quadrature-mixed to passband in the standard way. The real and imaginary components are first converted to the analog domain using digital-to-analog converters (DACs); the analog signals are then used to modulate cosine and sine waves at the carrier frequency, $f_c$, respectively. These signals are then summed to give the transmission signal, $s(t)$. The receiver picks up the signal, $r(t)$, which is then
quadrature-mixed down to baseband using cosine and sine waves at the carrier frequency. This also creates signals centered on $2f_c$, so low-pass filters are used to reject these. The baseband signals are then sampled and digitized using analog-to-digital converters (ADCs), and a forward FFT is used to convert back to the frequency domain. This returns $N$ parallel streams, each of which is converted to a binary stream using an appropriate channel encoding. These streams are then re-combined into a serial stream, $S(n)$, which is an estimate of the original binary stream at the transmitter[20].

III. OSCILLATOR NOISE ANALYSIS IN OFDM SYSTEMS

A. Phase and Amplitude Noise

An oscillator is a system that generates a periodic signal with a specified or controllable frequency. There are the two kinds of noise in a RF oscillator output, namely amplitude noise and phase noise. The output voltage of an oscillator is approximated by a power density function (PDF) over the channel bandwidth.

\[ V_{out}(t) = (A_0 + a_n(t)) \cos(2\pi f_0 t + \phi_n(t)) \]  
\[ V_{out}(t) = A(t) \cos(2\pi f_0 t + \phi(t)) \]  

Here $A_0$ is the average amplitude of the output signal and $f_0$ is nominal frequency of oscillation. $a_n(t)$ and $\phi_n(t)$ represent the time varying components of the amplitude and phase, respectively. These components are considered as noise because an ideal oscillator would have constant amplitude $A_0$ and phase varying at a constant rate $2\pi f_0$.

It can be shown that white noise power of the amplifier electronics is equally partitioned into amplitude and phase noise components with zero mean and $\sigma^2_a$, $\sigma^2_\phi$ variance respectively [13]. The SNR performances that can be achieved in an OFDM system with a noisy oscillator can be approximated by integrating its phase and amplitude noise power density function (PDF) over the channel bandwidth. The phase noise PDF of a Lorenzian function with uniform phase distribution. It is parameterized by its total integrated phase noise $K$ and $-3$dB bandwidth $B$, to which we superimpose a noise floor ($35$dB above the thermal noise) as shown in (3) [5]:

\[ L(f) = \frac{1}{\pi} \frac{K B}{f^2 + B^2 + L_0} \]  

(3)

Fig. 2 shows Phase noise PDF for different K and -3dB bandwidth of 100 Hz. The amplitude noise power density function of an oscillator is similar to phase noise power density function, but in average it is less than it in spectrum bandwidth, which $n$ for practical parameter of an oscillator is normally between 10 to 20 dB relative to oscillators quality [21]. Therefore the phase and amplitude noise variance are:

\[ \sigma^2_a = 2 \int \frac{L(f)}{f} df = 2 \frac{K B}{\pi} (f_2 + B^2 + L_0) \]  
\[ \sigma^2_\phi = \sigma^2_a \times 10 \frac{\pi}{2} \]  

(4)

Table 1 shows phase and amplitude noise variance of an oscillator for different $K$, $N$. In next section we use these values for calculating MQAM-OFDM degradation caused by phase and amplitude noise in simulation.

<table>
<thead>
<tr>
<th>$K$</th>
<th>$\sigma^2_a$</th>
<th>$\sigma^2_\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0010</td>
<td>0.0010</td>
</tr>
<tr>
<td>$n=100$</td>
<td>3.2e-13</td>
<td>3.2e-13</td>
</tr>
<tr>
<td>$n=20$</td>
<td>0.00001</td>
<td>0.00001</td>
</tr>
<tr>
<td>$n=15$</td>
<td>0.00001</td>
<td>0.00001</td>
</tr>
<tr>
<td>$n=10$</td>
<td>0.00001</td>
<td>0.00001</td>
</tr>
<tr>
<td>$n=5$</td>
<td>0.00001</td>
<td>0.00001</td>
</tr>
</tbody>
</table>

B. Theoretical Analysis

As shown in Fig. 1 if $N$ sub-carriers are used, and each sub-carrier is modulated using $M$ alternative symbols, the OFDM symbol alphabet consists of $M^N$ combined symbols. The low-pass equivalent OFDM signal is expressed as [1]:

\[ S(t) = \sum_{j=1}^{N} X_{j} e^{j 2 \pi f c t} \quad 0 \leq t < T \]  

(5)

where $j = \sqrt{-1}$, $X_{j}$ is the data symbol for the $k$ th sub-carrier, $N$ is the number of sub-carriers, and $T$ is the OFDM symbol time. The sub-carrier spacing of $1/T$ makes them orthogonal over each symbol period. $S(t)$ is corrupted by the phase and amplitude noise of the oscillator. Also, the received signal is influenced by the phase and amplitude noise of the RX local oscillator. It can be assumed that the phase and amplitude noises of the local oscillators at the transmitter and the receiver are independent and identical. So, it is expressed as follows:

\[ r(t) = (S(t) A_{XX}(t) e^{j \phi_{RX}(t)} + n(t)) A_{XX}(t) e^{j \phi_{RX}(t)} \]  
\[ Y_k = \frac{1}{N} \sum_{m=0}^{N-1} r(m) e^{-j 2 \pi k m / N} \]  
\[ Y_k = \frac{1}{N} \sum_{m=0}^{N-1} S(m) A_{XX}(m) e^{j \phi_{RX}(m)} e^{-j 2 \pi k m / N} + \sum_{m=0}^{N-1} n(m) e^{j \phi_{RX}(m)} e^{-j 2 \pi k m / N} \]  

(6)

\[ \sum_{m=0}^{N-1} A_{XX}(m) e^{j \phi_{RX}(m)} e^{-j 2 \pi k m / N} + \sum_{m=0}^{N-1} n(m) e^{j \phi_{RX}(m)} e^{-j 2 \pi k m / N} \]  

(7)
where \(n(t)\) is the complex Gaussian noise. \(\varphi_{xx}(t), A_{xx}(t)\) & \(\varphi_{rx}(t), A_{rx}(t)\) are the time-varying phase and amplitude noise processes generated by the RF local oscillator at the transmitter and the receiver. The sampled signal for the \(k\)th sub-carrier after FFT (fast fourier transform) processing stage in the receiver can be written as:

\[
Q_k = \text{FFT}(Y) = Y_k + X_k + N_k
\]

where:

\[
Q_k = \frac{1}{N} \sum_{n=0}^{N-1} A(n) e^{j\varphi(n)} e^{-j\frac{2\pi}{N} kn}
\]

or

\[
Y_k = Y_{k1} + Y_{k2} + N_k
\]

The received signal is composed of three contributions: \(Y_{k1}\) is the \(k\)th desired sub-carrier, \(Y_{k2}\) is the others and \(N_k\) is the AWGN, that is \([7],[9],[10],[13]-[15]:

\[
Y_k = Y_{k1} + Y_{k2} + N_k
\]

The expectation value of \(X_k Q_0\) is the desired signal component. So, the desired signal power can be found as:

\[
P_s\text{-desired} = |E[X_k Q_0]|^2 = P_s |E[Q_0]|^2
\]

where:

\[
E[Q_0] = \frac{1}{N} \sum_{n=0}^{N-1} EA(m) e^{j\varphi(m)} = \frac{1}{N} N . EA(m) e^{j\varphi(m)}
\]

\[
= EA(m) E[e^{j\varphi(m)}] = e^{-\sigma_r^2}
\]

\[
E[e^{j\varphi(m)}] = \frac{1}{\sqrt{2\pi} \sigma_r} e^{-\frac{\varphi^2}{2\sigma_r^2}} d\varphi = \frac{1}{\sqrt{2\pi} \sigma_r} e^{-\frac{\sigma_r^2}{2}}
\]

Therefore:

\[
P_s\text{-desired} = P_s e^{-\sigma_r^2}
\]

Variance of \(X_k Q_0\) is the phase and amplitude noise power caused by CPE(Common Phase Error) and \(P_{CPE}\) is calculated as follows:

\[
P_{CPE} = Var[X_k Q_0] = P_s Var[Q_0]
\]

where:

\[
Var[Q_0] = Var[\sum_{n=0}^{N-1} A(m)e^{j\varphi(m)}] + \frac{1}{N} \sum_{n=0}^{N-1} Var[A(m)e^{j\varphi(m)}]
\]

\[
= \frac{1}{N} (E[A(m)]^2 + |e^{j\varphi}|^2 - 2A^2 e^{-j\varphi}) = \frac{1}{N} (1 + \sigma_r^2 - e^{-\sigma_r^2})
\]

Therefore:

\[
P_{CPE} = P_s \times \frac{1}{N} (1 + \sigma_r^2 - e^{-\sigma_r^2})
\]

Next, \(\sum_{l=0}^{N-1} X_l Q_{l-1}\) is the interference component caused by ICI (Inter-sub-Carrier Interference). So, \(P_{ICL}\) can be calculated as:

\[
P_{ICL} = P_s \text{Var} \left[\sum_{l=0}^{N-1} Q_l\right] - \text{Var}[Q_0]
\]

where:

\[
\text{Var}[Q_0] = \frac{1}{N} \sum_{l=0}^{N-1} \text{Var} [A(m)e^{j\varphi(n)}e^{-j\frac{2\pi}{N} ln}]
\]

\[
= \frac{1}{N^2} \sum_{n=0}^{N-1} \text{Var} [A(m)e^{j\varphi(n)}e^{-j\frac{2\pi}{N} ln}] = (1 + \sigma_r^2 - e^{-\sigma_r^2})
\]

Therefore:

\[
P_{ICL} = P_s \left(1 - \frac{1}{N} (1 + \sigma_r^2 - e^{-\sigma_r^2})\right)
\]

Next, the above-mentioned powers are used to evaluate the performance degradation of the OFDM system in the presence of phase and amplitude noise of local oscillators. So, degradation factor (DF) is calculated as follows:

\[
DF = 10 \log \left(\frac{\text{SNR}_{\text{desired}}}{\text{SNR}_{\text{desired}}}\right) = 10 \log \left(1 + \sigma_r^2 - e^{-\sigma_r^2}\right)
\]

In (24), \(\sigma_r^2\) is the variance of random variable \(\varphi_{rx}\) and \(\varphi_{xx}\) and \(\sigma_r^2\) is the variance of random variable \(A_{rx}\) and \(A_{xx}\). These are calculated by integrating the power spectrum density of total output phase and amplitude noise.

IV. PERFORMANCE ANALYSIS

This paper presents performance analysis of OFDM communication system influenced by the phase and amplitude noise of the RF local oscillator in AWGN channel by both theoretical approach and simulation method. Phase and amplitude noise in the OFDM system is difficult to be compensated because of the strong dispersive component. Therefore, it is important to quantify the permissible level of phase and amplitude noise in local oscillator to maintain system performance and to design local oscillator with limited maximum phase and amplitude.
noise. The BER formula for system performance analysis can be found by the SNR with phase and amplitude noise instead of the SNR without phase and amplitude noise. Equations (25)-(26) shows probability of bit-error per carrier and probability of bit-error for QAM modulation respectively[22]. Where k is number of bits per symbol, M is number of symbols in modulation constellation, $E_b$ is energy-per-bit and $N_0$ is noise power spectral density.

$$P_{bc} = \frac{2}{k} \left( 1 - \frac{1}{\sqrt{M}} \right) \left( \frac{3k}{M-1} \right) \left( \frac{E_b}{N_0} \right)$$

(25)

$$P_b = 1 - (1 - P_{bc})^2 = 2P_{bc} - P_{bc}^2$$

(26)

As shown in Fig. 3-4 and Table II the relative SNR penalty (Degradation) to meet BER=10^{-6} for different amplitude noise order, n, represented in Table III. The results from Tables II-III shows that if the amplitude noise order relative to phase noise, n, was higher than 15 dB (15-20 dB), we can ignore the amplitude noise effect in performance analysis but if it was less than 15 dB (10-15 dB) we must consider the amplitude noise for exact analysis and determining the tolerable level of amplitude noise.

Fig. 5 shows the combination of two effects (CPE, ICI) on a 4-QAM-OFDM signal after demodulation caused by phase and amplitude noise. The first scatter plot (a) corresponds to the demodulated signal when the variance of the phase noise that affects the OFDM signal is zero. The second scatter plot (b) corresponds to the demodulated signal when the variance of the phase noise is 0.0316 and amplitude noise order relative to phase noise is n=100dB (ideal) and The third scatter plot (c) corresponds to the demodulated signal when the variance of the phase noise is 0.0316 and amplitude noise order relative to phase noise is n=10dB. As shown in Fig.5 clouds in the constellation of (c) has the higher spreading tendency and cause higher performance degradation.
the 4, 16, 64, 256 QAM modulation format.

Simulation results of the OFDM communication system of results by theoretical approach closely match with the phase and amplitude noise effects on OFDM signals has been presented. Results can be arranged as follows:

1. Since an accurate prediction of the tolerable phase noise variance to meet different BER for different amplitude noise order, $n$, is quantified. 3- The amplitude noise source of RF oscillator has a little effect on performance degradation of M-QAM OFDM system when it is between 15-20 dB, but when is between 10-15 dB we must consider it for exact performance degradation analysis.

**REFERENCES**


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**TABLE II: PHASE AND AMPLITUDE NOISE DEGRADATION IN 4, 16, 64,256 QAM-OFDM FOR BER=10^-6.**

<table>
<thead>
<tr>
<th>$\sigma_p^2$</th>
<th>$\sigma_e^2$</th>
<th>$\sigma_{p_e}$</th>
<th>$\sigma_{e_p}$</th>
<th>$\sigma_{p_e}$</th>
<th>$\sigma_{e_p}$</th>
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</thead>
<tbody>
<tr>
<td>$P_e$</td>
<td>Modulation</td>
<td>0.0134</td>
<td>0.0032</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>4QAM</td>
<td></td>
<td>6.083</td>
<td>1.219</td>
<td>0.351</td>
<td>0.164</td>
</tr>
<tr>
<td>16QAM</td>
<td></td>
<td>0.321</td>
<td>1.332</td>
<td>0.534</td>
<td>0.193</td>
</tr>
<tr>
<td>64QAM</td>
<td></td>
<td>6.590</td>
<td>1.280</td>
<td>0.262</td>
<td>0.107</td>
</tr>
<tr>
<td>256QAM</td>
<td></td>
<td>7.569</td>
<td>1.348</td>
<td>0.386</td>
<td>0.113</td>
</tr>
</tbody>
</table>

**TABLE III: RELATIVE DEGRADATION FOR DIFFERENT AMPLITUDE NOISE ORDER IN M-QAM-OFDM FOR BER=10^-6.**

<table>
<thead>
<tr>
<th>$\sigma_p^2$</th>
<th>$\sigma_e^2$</th>
<th>$\sigma_{p_e}$</th>
<th>$\sigma_{e_p}$</th>
<th>$\sigma_{p_e}$</th>
<th>$\sigma_{e_p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_e$</td>
<td>Modulation</td>
<td>0.0136</td>
<td>0.0032</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>10^4</td>
<td></td>
<td>0.0136</td>
<td>0.0032</td>
<td>0.001</td>
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</tbody>
</table>

![Fig. 5. Constellation point of 4QAM signal](image-url)
M. H. Madani was born in Iran in 1976. He received the B.Sc., M.Sc and Ph.D degrees in Electrical Engineering in 1998, 2001 and 2010, respectively. His research areas include wireless communication systems. He is currently a member of Microwave/ mm-Wave and Wireless Communication Research Lab in Electrical Engineering Department of Amirkabir University of Technology.

A. Abdipour was born in Alashtar, Iran, in 1966. He received the B.Sc. degree in electrical engineering from Tehran University, Tehran, Iran, in 1989, the M.Sc. degree in electronics from Limoges University, Limoges, France, in 1992, and the Ph.D. degree in electronics from Paris XI (Orsay) university, Paris, France, in 1996. He is currently a Professor with the Electrical Engineering Department, Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran. He authored three books about Noise in Electronic Communication Circuits and Systems and Transmission Lines. His research areas include wireless communication systems (RF technology and transceivers), RF/microwave/mm-wave circuit and system design, electromagnetic (EM) modeling of active devices and circuits, high-frequency electronics (signal and noise), nonlinear modeling, and analysis of microwave devices and circuits. He has authored or coauthored over 200 papers in refereed journals and local and international conferences.

A. Mohammadi received the B.Sc. degree in Electrical Engineering from Tehran University, Iran in 1988, the M.Sc. and Ph.D. degrees in Electrical Engineering from the University of Saskatchewan, Canada, in 1995, and 1999, respectively. He was a researcher at Telecommunications Research Lab (TRLabs), Canada, from 1995 to 1998. In 1998, he joined to Vecima Networks Inc., Microwave Research Lab, Victoria, Canada, as a senior research engineer where he conducted research on Microwave and Wireless Communications. Since March 2000, he has been with the Electrical Engineering Department of Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran. In 2008, he joined to Electrical and Computer Engineering Department of the University of Calgary, Canada as a visiting professor. He has published over 120 Journal and Conference papers and holds three U.S. and one Canadian Patents. His current research interests include broadband wireless communications, adaptive modulation, MIMO Systems, Mesh and AdHoc Networks, Microwave and Wireless Subsystems, and direct conversion transceivers. Currently he is Director of Radiocommunication Center of Excellence of Amirkabir University of Technology (Tehran Polytechnic).