Narrow-Scope Searching FrFT Method Applied in the Elimination of Sonar Ranging Error

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Abstract—For the question that the computation of searching peak is large in signal parameters estimation using FrFT method, a new method of narrow-scope searching FrFT is proposed in this paper. According to the possible speed range of the target’s motion, the change range of received LFM signal’s parameters caused by Doppler Effect can be predicted. By estimate the FrFT optimal order in a very small range accurately, the search scope and computation amount will be greatly reduced. And this method was successfully applied to eliminate the active sonar ranging error. Theoretical analysis and simulation have confirmed small operation amount, high accuracy and good real-time performance for this presented method.

Index Terms—Active sonar, error elimination, fractional fourier transform (FrFT), linear frequency modulation (LFM) and parameter estimation.

I. INTRODUCTION

The time length and broadband of Linear Frequency Modulation (LFM) signal can adjust individually. It is a complex and compressible signal, and has widely used in radar, sonar, Acoustic Doppler Current Profilers (ADCP) [1]-[3] and other systems. The statistical properties of LFM signal change with time, then it is a typical of non-stationary signal compared to the traditional Fourier transform. With the development of non-stationary signal processing theory, a variety of time-frequency analysis methods are emerging, such as short time Fourier transform (STFT), wavelet transform (WT) [4], Wigner-Ville Distribution (WVD) [5], WVD-Hough Transform (WHT) [6], [7], Fractional Fourier Transform (FrFT) [8] and so on. The observation window of STFT is very narrow, and the width of the WT’s time window is varied. These characteristics all affect the signal resolution in the time-frequency domain. The WVD used on the LFM signal has a good time-frequency gathered performance, but because it is non-linear transformation, when dealing with multi-component LFM signal it will be affected by cross-term then the performance lowing [9]. WHT can effectively suppress cross-term interference, but the calculation consumes too much time and the initial phase cannot be estimated. FrFT is a linear transformation, without cross-term interference, and is very suitable for processing the LFM signal. The traditional Fourier transform can be seen as signal decomposition on harmonic wavelet, when the signal is a harmonic component, the energy of the Fourier transform will gather on the corresponding harmonics frequency. Similar to the Fourier transform, FrFT also can be seen as signal decomposition based on the Chirp, when the LFM of the order with the FrFT into a certain relationship, when the LFM modulation frequency and FrFT order have a certain relationship, the energy of conversion results will focus on a particular point.

In the propagation of the acoustic wave, the phenomenon that the signal frequency shifted due to the Doppler Effect is known as the Doppler frequency shift. There are two main reasons for Doppler shift is generated on underwater acoustic channel, one is the relative movement of the signal received and transmitted and the other is the movement of the media. Because of the Doppler shift, the received signal of LFM signal has changed in the time scale transformation. When the center frequency, wavelength and phase of LFM signal changed, the DOA estimation accuracy will be inevitably affected.

Based on the above, a compensation method of the Doppler shift mismatch in the active sonar is proposed. The parameter of the echo signal is estimated by FrFT in order to adjust the parameters of the pulse compression signal and reduce the adverse effects of the Doppler frequency shift. In order to reduce the computation and improve the real-time, a method that reduces FrFT peak search range according to the possible velocity range of the target body is proposed. The simulation results show that the mismatch compensation effect is obvious, and the computation is reduced a lot than common FrFT, and the practicality of the method is obvious.

II. ANALYSIS OF THE IMPACT OF DOPPLER SHIFT ON LFM PULSE COMPRESSION

LFM pulse signal with noise can be expressed as:

$$x(t) = a_o \text{rect}(\frac{t}{T}) \exp(j\phi_0 + j2\pi f_0 t + j\pi \mu t^2) + w(t) \quad (1)$$

Wherein, respectively, $a_o$, $\phi_0$, $f_0$ and $\mu = B/T$ is the amplitude, the initial phase, the center frequency and the modulation frequency of the LFM signal. $\text{rect}(t/T)$ is a T-width rectangle function and $w(t)$ is an additive white Gaussian noise.

When there is relative movement between the measuring probe and the target device, the frequency of the received signal will change and the center frequency also changes. For a single-frequency signal $f_s$, the moving speed of transmitting side is assumed as $v_x$, the moving speed of the
receiving end is assumed as $v_R$. The frequency $f_0$ of the received signal can be calculated by (2).

$$f_R = k f_T, \quad k = \frac{c + V_x}{c - V_R}, \quad f_d = (k - 1) f_T$$

Wherein, $V_x, V_R$ are vectors, and they take transmit to the receiver side as the positive direction, their units are m/s. The units of $f_T$ and $f_R$ are Hz, $c$ stands for the sound speed in seawater, and generally 1500 m/s [10], the coefficient $k$ is defined as ratio between the received signal frequency and the frequency of the transmitted signal, called the Doppler factor. $f_d$ is the Doppler shift frequency.

Equation (2) shows that scale transformation on time domain occurred on the LFM pulse signal affected by Doppler Effect. Assume that the transmission signal is $x(t)$ shown in (1), the Doppler Effect makes the signal becomes $x(kt - \tau)$ at the receiving end. Wherein, $k$ is the Doppler factor, $\tau$ is the receiving delay. The center frequency of received signal becomes $kf_0$ from $f_0$, frequency modulation rate becomes $k^2 \mu$ from $\mu$.

In active sonar systems, array element spacing is assumed fixed $d$, the DOA of an acquisition target is $\theta$, and the delay of element $i$ remains at $\tau_i$. The phase difference $\phi_i$ between two adjacent array elements as (3) shows. Doppler Effect makes the phase difference of received signal between two adjacent array elements from $\phi_i$ to $k \phi_i$.

$$\phi_i = -2 \pi \sin \theta \left( (i-1) d / c \right) f_0$$

The signal after matched filter of a LFM pulse signal without Doppler Effect shows in (4) [10]. On the contrary, a LFM pulse signal with Doppler Effect becomes $y(t - f_d / \mu)$ after the matched filtering. Wherein, $f_d$ stands for the Doppler shift frequency and $\mu$ is the frequency modulation rate of LFM signal. In other words, Doppler Effect makes the signal envelope after pulse compression shifts $\Delta t = f_d / \mu$ in the timeline. Ranging error caused by this is $\Delta S = c \Delta t / 2 = c f_d / (2 \mu)$.

$$y(t) = a_0 \sqrt{BT} \sin \left[ \pi B (t - t_d) \right] / \pi B (t - t_d)$$

In (4), $a_0$ is pulse amplitude, $B$ stands for pulse width, $T$ is FM width and $t_d$ is the additional delay caused by match filter.

### III. COMPENSATE RANGING ERROR USING FrFT

#### A. FrFT’s Energy Concentration Characteristics on LFM Signal

FrFT is a generalized form of Fourier transform. If Fourier transform is a signal rotate counterclockwise $\pi/2$ from the time axis to the frequency axis in the time-frequency plane, FrFT can be regarded as a signal rotate counterclockwise angle $\alpha$ from the time axis to the $u$ axis. P-order FrFT of signal $x(t)$ is defined as (5).

$$X_p(u) = F^p[x(t)] = \int_{-\infty}^{\infty} K_p(u,t)x(t)dt$$

Wherein, $p \in R$ is the order of FrFT, $F^p[*]$ is FrFT operator and $K_p(u,t)$ is FrFT’s kernel function.

$$K_p(u,t) = \left[ A_j \exp \left[ j \pi \left( u^2 \beta - 2 u \gamma + \tau^2 \beta \right) \right] \right]_{\alpha \neq n \pi}$$

In (6), $\beta = \cot \alpha \ , \ \gamma = \csc \alpha \ , \ \alpha = \pm \pi / 2 (p \in R)$ and $A_{\alpha} = \sqrt{-j \cot \alpha}$. FrFT can be seen as the decomposition of a signal based on Chip bases. When the frequency modulation rate of a LFM signal and FrFT order have a certain relationship, the energy of conversion results will focus on a particular point [11]. For a noise-free LFM signal $x(t) = a_0 \exp(j 2 \pi f_0 t + j \pi \mu t^2)$, when its center frequency meets $f_0 = 0$ and the FrFT order meets $p = 2 \arccos(-\mu) / \pi$, it’s FrFT is defined by (7).

$$F^p[x(t)] = X_p(u) = A_\alpha \delta(u)$$

In (7), $A_p$ is a $p$-related factor, $\delta(u)$ is impulse function. Formula (7) shows that the conversion energy approximately focuses on a point. This $p$ is called the optimal order.

#### B. Compensation Method of Ranging Error

As shown in Section II, the Doppler Effect will generate a target ranging error $\Delta S = \mu f_d / 2$. To this end, an error compensation method is proposed: to estimate the center frequency and FM rate of received LFM pulse signal using FrFT, then use them to correct the matched filtering signal’s parameters in real time, in order to reduce the ranging error.

#### C. Narrow-Scope Searching FrFT Method

A LFM signal with noise can be defined by the (8).

$$x(t) = a_0 \exp(j \phi_0 + j 2 \pi f_0 t + j \pi \mu t^2) + w(t)$$

Wherein, respectively, $a_0, \phi_0, f_0$ and $\mu = B / T$ is the amplitude, the initial phase, the center frequency and the modulation frequency of the LFM signal. The process of detecting and estimating above LFM signal using FrFT method can be described as (9) and (10).

$$\hat{\delta}_0, \hat{\theta}_0 = \arg \max_{\delta, \theta} \left| X_{\alpha_0}(u) \right|^2$$

$$\hat{\phi}_0 = \arg \left[ \frac{X_{\alpha_0}(\hat{\theta}_0)}{A_{\alpha_0} \exp(j \pi \hat{\theta}_0 \cot \hat{\alpha}_0)} \right]$$

$$\hat{\theta}_0 = \left| X_{\alpha_0}(\hat{\phi}_0) \right| / \Delta |A_{\alpha_0}|$$

In order to reduce computation and improve this method’s usability, we narrow FrFT’s peak searching range in a small scope $\{\alpha_{\min}, \alpha_{\max}\}$ which can be estimated by the
possible region of \([-v_{m}, v_{m}]\) relative velocity between the detection equipment and test objectives. Comparing to the full-scope searching FrFT method, the narrow-scope searching method can exponentially improve the estimation accuracy in the case of ensuring the same amount of computation, also can also exponentially reduce the amount of computation in the case to ensure the same estimation accuracy.

Firstly, get the range of Doppler factor \(k\) by \(k = (c-v_T)/c-v_R\) and \(v_T - v_R \in [-2v_m, 2v_m]\) shown in (2).

\[
\frac{c - v_m}{c + v_m} \leq k \leq \frac{c + v_m}{c - v_m}
\]  

(11)

Then, to deduce the region of received signal’s FM rate according to the conclusion shown in the Section II: FM rate of received signal is \(k^2\) times transmitted signal’s FM rate.

\[
\left(\frac{c - v_m}{c + v_m}\right) \mu \leq \hat{\mu} \leq \left(\frac{c + v_m}{c - v_m}\right) \mu
\]  

(12)

Before doing FrFT of a LFM signal, this LFM signal should be dimensionless normalized. There is a relationship between signal frequencies and FM rates before and after normalization [12].

\[
f_{0}' = f_{0}\sqrt{f_T/f_s}, \mu' = \mu f_T/f_s
\]  

(13)

In (13), \(f_{0}'\) is normalized frequency and \(\mu'\) is normalized FM rate. Therefore, the forecasting range of normalized FM rate shows as follows.

\[
\left(\frac{c - v_m}{c + v_m}\right) \frac{\mu f_T}{f_s} \leq \hat{\mu}' \leq \left(\frac{c + v_m}{c - v_m}\right) \frac{\mu f_T}{f_s}
\]  

(14)

Then, according to the (10) and the first line of (12), the search range of FrFT’s rotation angle \(\alpha\) is derived as follows.

\[
\arccot\left[-\left(\frac{c + v_m}{c - v_m}\right) \frac{\mu f_T}{f_s}\right] \leq \alpha \leq \arccot\left[-\left(\frac{c - v_m}{c + v_m}\right) \frac{\mu f_T}{f_s}\right]
\]  

(15)

The calculation complexity of (9) is greatly reduced due to \(c\geq v_{m}\) and the search range the rotation angle \(\alpha\) in (15) is limited to a small range. Here is an example, assuming \(c=1500m/s\), \(v_m=30m/s\) and \(\mu=1000Hz/s\), the range of rotation angle \(\alpha\) can be calculated as \(4.7353 \leq \alpha \leq 4.7392\) or \(1.5937 \leq \alpha \leq 1.5977\). The presence of two intervals is due to the periodic of the anti-cotangent function. In case of ensuring same searching accuracy, the calculation complexity of this method is only about 1/60000 full-scope searching method (0 ≤ \(\alpha\) ≤ 2\(\pi\)).

Discrete FrFT can be calculated by using Ozaktas algorithm [13] in practical engineering applications. This algorithm can be implemented using FFT, and it has the same computational complexity \(O(N \log N)\) with FFT algorithm. Before calculating, the signal should necessarily be dimensionless normalized. In order to make the range of signal’s time domain representation is defined on \([-T/2, T/2]\) and the range of its frequency domain representation is defined on \([-F/2, F/2]\). The form of discrete FrFT calculation can be defined as (16).

\[
X_n(m) = F^n[x(nT/F)]
\]  

(16)

Wherein, \(X_n(m)\) is the DFrFT of signal \(x(t)\). \(\gamma = \cot \alpha\), \(\beta = \csc \alpha\), \(\alpha = \pi / 2\).

D. Correction of Matched Filtering Parameters

Basing the (9) and (10) in Section A, we can estimate the frequency and FM rate of received signal, and the estimated values respectively are \(\hat{f}_{0}\) and \(\hat{\mu}\). The relationship between them and the transmitted signal’s ones are shown as follows.

\[
\hat{f}_{0} = k f_{0}, \hat{\mu} = k^2 \mu
\]  

(17)

In the case of none compensation for the ranging error, the frequency and FM rate of the copy signal during the matched filtering of received signal are respectively \(f_{0}\) and \(\mu\). Due to the Doppler Effect, an error \(\Delta S = c f_{0} / (2\mu)\) is generated during the active sonar ranging. To eliminate this error, frequency and FM rate of a copy signal using in matched filtering are adjusted to \(\hat{f}_{0}\), \(\hat{\mu}\).

The theoretical analysis in the Section II of this paper shows that this method can effectively eliminate ranging error. Meanwhile, because of the small amount of calculation, parameter adjustment can be done in real time.

IV. SIMULATION ANALYSIS

In order to verify the validity and efficiency of estimating a LFM signal’s center frequency and FM rate using narrow-scope searching FrFT method, as well as the validity of correcting the ranging error caused by the Doppler effect using matching parameters correction method, the following simulation analysis is given.

In an active sonar system, the transmitted signal is a LFM pulse signal, pulse width is \(T = 0.5s\), sampling frequency \(f_s = 20000 Hz\), the number of sampling points is 10001. Center frequency is \(f_{0} = 500Hz\), FM rate is \(\mu = 1000Hz/s\). Interference is Gaussian white noise, SNR is 0dB. Assuming measurement equipment is fixed and measuring target frontally approaches at the speed of 60 knots (30m/s). Doppler factor \(k = 1.0208\) can be calculated by using the equation (2). Theory values of received signal’s frequency and FM rate respectively are \(f_{0} = 510Hz\) and \(\mu = 510Hz/s\).

Before FrFT calculation for the received signal, it needs to have the received signal be dimensionless normalized. Process is as follows: Firstly, set the time origin at the midpoint of signal’s observation time to make the signal’s time-domain locates on the interval \([-0.25s, 0.25s]\), frequency-domain locates on the interval \([-10000Hz, 10000Hz]\), so the scale factor \(S = \sqrt{f_T/f_s} = 0.005s\) and the normalized width [11] \(x = \sqrt{f_T/f_s} = 100\). Secondly, change the signal’s time-domain to the interval \([-0.25/ S, 0.25/ S]\),
change the sampling rate to \( x \cdot f_0 \cdot S \) and change the FM rate to \( \mu \cdot S \). Both time-domain and frequency-domain of the normalized signal locate on the interval \([-x/2, x/2]\) i.e., \([-50, 50]\).

According to the analysis in Section III-B Part A, the forecasting range of FrFT optimum rotation angle for the received signal is \( 4.7353 \leq \alpha \leq 4.7392 \) or \( 1.5937 \leq \alpha \leq 1.5977 \) (double interval is due to periodic of anti-cotangent function), corresponding to the forecasting range of the order \( p \) is \( 3.015 \leq p \leq 3.017 \) or \( 1.015 \leq p \leq 1.017 \). The comparison of two FrFT methods’ simulation results are shown as follows.

Fig. 1. The contrast of two FrFT methods’ estimation result on same signal.

Firstly, do full-scope searching FrFT whose parameters are \( 0 \leq p \leq 4 \) and stepper = 0.001 for the received LFM signal, the simulation result is shown in Fig. 1. The evaluation result is \( \hat{p} = 3.017 \) and \( \hat{\mu} = -2.54 \). The time this computation consumed is \( t_f = 77.1668s \). The estimation values of the analyzed signal’s center frequency and FM rate before normalization respectively are \( \hat{f}_0 = 508 \text{ Hz}, \) \( \hat{\mu} = 1068 \text{ Hz/s} \) according to (10) and (13).

Fig. 2. Ranging error compensation by updating matched filter’s parameters.

Secondly, do narrow-scope searching FrFT whose parameters are \( 3.015 \leq p \leq 3.017 \) and stepper = 0.001 for the
received LFM signal, the simulation result is shown in Fig. 2. The evaluation result also is $\hat{\mu} = 3.017$ and $\hat{u} = -2.54$. The time this computation consumed is $t_\text{cpu} = 0.0757$s. The estimation values of the analyzed signal’s center frequency and FM rate before normalization respectively are $\hat{f}_0 = 508$ Hz, $\hat{\mu} = 1068$ Hz/s according to (10) and (13).

The evaluation results of narrow-scope searching FrFT method and full-scope searching FrFT are same. But the former’s time spent is only one-thousandth the latter’s.

Thirdly, update the copy signal’s center frequency and FM rate during matched filtering using the estimation values of the received signal’s parameters. Fig. 2 shows the contrast of matched filtering results between the no-update and update situations.

It’s easy to see, when the parameters are NOT updated, there is an obvious time delay between the matched filtering results and theoretical waveforms on the timeline. The delay is the reason for ranging error. Differently, when the parameters are updated, there is no time delay basically between the matched filtering results and theoretical waveforms on the timeline. The ranging error is compensated successfully.

V. Conclusion

This paper focuses on using the narrow-scope searching FrFT method to estimate the parameters of LFM signal and compensating the active sonar’s ranging error caused by the Doppler Effect by updating matched filter’s parameters with estimation values in real time. Theoretical analysis and simulation have both confirmed that the narrow-scope searching FrFT method can estimate LFM signal’s parameters accurately, and its computational amount reduces three orders of magnitude at least than the full-scope searching FrFT method. Update matched filter’s parameters method can compensate meter-level ranging error of active sonar system basing on signal parameters’ accurate estimation on FrFT method. Good characteristics of having small computation amount and good real-time performance make narrow-scope searching FrFT method has high value in engineering applications.

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