Blind Equalization Using Correntropy Algorithm for Wireless Sensor Networks

Namyong Kim and Ki-Hyeon Kwon

***Abstract*—Wireless links in indoor sensor networks have distortions due to multipath fading from reflections and impulsive noise from indoor electric devices. In these harsh environments, blind correntropy equalization algorithm yields superior MSE performance compared with the constant modulus algorithm. However the correntropy algorithm has a heavy computational complexity, which is not suitable for power and cost effectiveness demanded in wireless sensor networks. In this paper, a new gradient estimation for weight updates of the correntropy algorithm in order to reduce its computational burden is proposed. For the size of the data block, *N-M*+1 including the number of lags *M*, the conventional correntropy algorithm requires (*N*+1)*M* − (*M*+1)*M*/2 multiplications, whereas the proposed method of recursively estimating the gradient does only $2M$. The simulation results show that the conventional and proposed gradient estimation methods yield exactly the same estimation traces proving justification of the proposed estimation. These results indicate that the proposed method can be implemented in reliable and efficient indoor sensor networks.**

***Index Terms*—Complexity, correntropy, impulsive noise, sensor network.**

I. Introduction

Wireless sensor networks require signal processing for spatially distributed sensors and wireless communication problems in addition to electronic control of sensors and actuators. Among the harsh problems in sensor network communication environment are multipath propagation [1].

Equalization is a powerful technique to compensate such multipath fading problems. Two types of equalization methods are used according to their purposes as training-aided equalization method and blind method. In wireless sensor networks in which sensors usually work with low duty-cycle training-aided equalizers need a sufficiently long training sequence in each data packet [2]. The training-aided equalizer algorithms are not suitable since equalizers for wireless sensor networks require the function of mitigation of multipath propagation and the efficiency of bandwidth, energy and cost as well [3].

Blind equalizer algorithms requiring no training sequences are appropriate for power and bandwidth efficiency [3]. The well-known constant modulus algorithm (CMA) has been developed for blind equalization based on the minimization of the instant error power defined as the difference between the output power and a constant modulus predefined according to the modulation schemes [4]. The CMA is known to work well in the environment of Gaussian noise but not of impulsive noise because it induces large instantaneous system errors that often make the CMA fail [5]. Indoor wireless sensor networks are interfered with impulsive noise from a various sources of impulse noise as well as Gaussian background noise [6].

Unlike the CMA that utilizes error power, the correntropy function has been proposed as a criterion of information theoretic learning to cope with impulsive noise problems and to enhance face recognition [5]-[8]. The correntropy blind algorithm developed based on the minimization of correntropy differences between source and equalizer output is known to be effective in impulsive noise contaminated environments.

However, the correntropy blind algorithm has a disadvantage of heavy computational complexity due to the double summation operations carried out at each iteration time in the weight update process. This computational burden makes the correntropy blind algorithm inappropriate in wireless sensor networks since computationally efficient techniques must be used for reliable and efficient signal demodulation and detection in sensor networks [3].

In this paper, for the purpose of the efficient implementation of the correntropy algorithm, a new method of reducing the computational complexity of the conventional correntropy algorithm while keeping the robustness of the algorithm to multipath and impulsive noise problems is proposed.

II. System Model and Correntropy Algorithm

In wireless sensor networks, the sensor signal is preprocessed to become sensor data suitable for transmission. Then the sensor data a_k at symbol time k are transmitted through the wireless multipath channel and impulsive noise n_k is added as described in baseband model in Fig. 1. The impulsive noise-added signal is received at the equalizer structured with a tapped delay line (TDL) with L weights. The wireless multipath channel can be expressed as $\sum h_i \delta(k-i)$ and the equalizer input becomes $x_k = \sum h_i a_{k-i} + n_k$ [3]. With the input vector $\mathbf{X}_k = [x_k, x_{k-1}, ..., x_{k-L+1}]^T$ and weight vector $\mathbf{W}_k = [w_{0,k}, w_{1,k}, .., w_{L-1,k}]^T$ the TDL equalizer produces the output $y_k = \mathbf{W}_k^T \mathbf{X}_k$. In many blind equalization schemes the difference $e_{CME,k}$ between the instant output power $|y_k|^2$ and

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a constant modulus $R_2 = E[|a_i|^4]/E[|a_i|^2]$ is to be minimized according to the MSE criterion P_{MSE} described in (1) and (2) [4].

$$e_{CME,k} = |y_k|^2 - R_2 \tag{1}$$

$$P_{MSE} = E[|e_{CME,k}|^2] \tag{2}$$

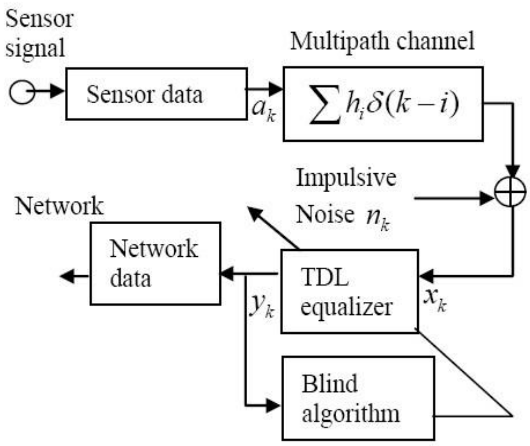

Fig. 1. Wireless transmission between a sensor node and its related network.

The CMA commonly being used in most blind equalization has been designed by minimizing the instant error power $e^2_{CME,k}$ instead of minimizing (2) for practical reasons. Impulsive noise can lead algorithms based on the instant error power to instability.

As a new correlation function for information theoretic learning (ITL), the correntropy function $V_Y[m]$ with lag m has been proposed as in (3) [5].

$$V_Y[m] = \frac{1}{N-m+1} \sum_{i=k-N+m}^{k} G_\sigma(y_i - y_{i-m}) \tag{3}$$

where $N-m+1$ is the size of the data block $\{y_k, y_{k-1}, \ldots, y_{k-N+1}, \ldots, y_{k-N+m}\}$. For blind equalization applications, the correntropy distance P_{CD} between the source correntropy $V_S[m]$ and the equalizer output correntropy $V_Y[m]$ is to be minimized [5].

$$P_{CD} = \sum_{m=1}^{M} (V_S[m] - V_Y[m])^2 \tag{4}$$

where M is the number of lags.

For the minimization of P_{CD} with respect to the TDL equalizer weight, the steepest descent method with the convergence parameter μ can be employed as

$$\mathbf{W}_{k+1} = \mathbf{W}_k - \mu \frac{\partial P_{CD}}{\partial \mathbf{W}} \tag{5}$$

The gradient $\frac{\partial P_{CD}}{\partial \mathbf{W}}$ in (5) is estimated using the data block $\{y_k, y_{k-1}, \ldots, y_{k-N+1}, \ldots, y_{k-N+m}\}$ as

$$\frac{\partial P_{CD}}{\partial \mathbf{W}} = \sum_{m=1}^{M} \sum_{i=k-N+m}^{k} \frac{1}{(N-m+1)\sigma^2} (V_S[m] - V_Y[m]) \cdot G_\sigma(y_i - y_{i-m}) \cdot (y_i - y_{i-m})(\mathbf{X}_i - \mathbf{X}_{i-m}) \tag{6}$$

Then the correntropy algorithm is obtained as follows [5].

$$\mathbf{W}_{k+1} = \mathbf{W}_k - \mu \sum_{m=1}^{M} \sum_{i=k-N+m}^{k} \frac{1}{(N-m+1)\sigma^2} (V_S[m] - V_Y[m]) \cdot G_\sigma(y_i - y_{i-m}) \cdot (y_i - y_{i-m})(\mathbf{X}_i - \mathbf{X}_{i-m}) \tag{7}$$

The correntropy algorithm is known to have the immunity to impulsive noise as well as the ability to compensate for the channel distortion from multipath fading [5]. However, it has a disadvantage of the heavy computational complexity due to the double summation operations in (6) or (7) at each iteration time. This computational burden prevents its implementation in wireless sensor networks demanding power and cost efficiency. Aiming at the efficient implementation of the correntropy algorithm, a method of reducing the computational complexity of the correntropy algorithm is proposed in the following subsection.

III. Proposed Algorithm Using Recursive Gradient Estimation of the Correntropy Distance

The conventional gradient estimation of (6) is considered as a block processing method using a data block of $\{y_k, y_{k-1}, \ldots, y_{k-N+1}, \ldots, y_{k-N+m}\}$ at each iteration time. In this section we present a computation-reduced estimation method by investigating how the next time gradient $\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k+1}$ is related with the current gradient $\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k}$. Let us define $\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k}^{I}$ and $\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k}^{S}$ as the initial state gradient for $1 \le k \le N$ and the steady state gradient for $k > N$, respectively. Then $\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k}^{I}$ and $\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k}^{S}$ can be expressed as

$$\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k}^{I} = \sum_{m=1}^{M} \sum_{i=1}^{k} \frac{1}{(k-m+1)\sigma^2} (V_S[m] - V_Y[m]) \cdot G_\sigma(y_i - y_{i-m}) \cdot (y_i - y_{i-m})(\mathbf{X}_i - \mathbf{X}_{i-m}) \tag{8}$$

$$\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k}^{S} = \sum_{m=1}^{M} \sum_{i=k-N+m}^{k} \frac{1}{(N-m+1)\sigma^2} (V_S[m] - V_Y[m]) \cdot G_\sigma(y_i - y_{i-m}) \cdot (y_i - y_{i-m})(\mathbf{X}_i - \mathbf{X}_{i-m}) \tag{9}$$

Firstly, the steady state gradient is investigated the relationship between the next time gradient and the current gradient. The next time gradient $\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k+1}^{S}$ becomes

$$\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k+1}^{S} = \sum_{m=1}^{M}\sum_{i=k+1-N+m}^{k+1}\frac{1}{(N-m+1)\sigma^2}(V_S[m] - V_Y[m])\cdot G_\sigma(y_i - y_{i-m})\cdot(y_i - y_{i-m})(\mathbf{X}_i - \mathbf{X}_{i-m}) \qquad (10)$$

Then we can divide (10) into terms related with the next iteration time $i = k+1$ and the remaining.

$$\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k+1}^{S} = \sum_{m=1}^{M}\sum_{i=k+1-N+m}^{k}\frac{1}{(N-m+1)\sigma^2}(V_S[m] - V_Y[m])\cdot G_\sigma(y_i - y_{i-m})\cdot(y_i - y_{i-m})(\mathbf{X}_i - \mathbf{X}_{i-m}) + \sum_{m=1}^{M}\frac{1}{(N-m+1)\sigma^2}(V_S[m-V_Y[m])\cdot G_\sigma(y_{k+1} - y_{k+1-m})\cdot(y_{k+1} - y_{k+1-m})(\mathbf{X}_{k+1} - \mathbf{X}_{k+1-m}) \quad (11)$$

Similarly, the equation (11) can be divided into terms related with the past iteration time $i = k+1-N+m$ and the remaining parts as

$$\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k+1}^{S} = \sum_{m=1}^{M}\sum_{i=k-N+m}^{k}\frac{1}{(N-m+1)\sigma^2}(V_S[m] - V_Y[m])\cdot G_\sigma(y_i - y_{i-m})\cdot(y_i - y_{i-m})(\mathbf{X}_i - \mathbf{X}_{i-m}) + \sum_{m=1}^{M}\frac{1}{(N-m+1)\sigma^2}(V_S[m-V_Y[m])\cdot G_\sigma(y_{k+1} - y_{k+1-m})\cdot(y_{k+1} - y_{k+1-m})(\mathbf{X}_{k+1} - \mathbf{X}_{k+1-m}) - \sum_{m=1}^{M}\frac{1}{(N-m+1)\sigma^2}(V_S[m-V_Y[m])\cdot G_\sigma(y_{k+1-N+m} - y_{k+1-N})\cdot(y_{k+1-N+m} - y_{k+1-N})\cdot(\mathbf{X}_{k+1-N+m} - \mathbf{X}_{k+1-N})$$

$$= \left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k}^{S} + \sum_{m=1}^{M}\frac{1}{(N-m+1)\sigma^2}(V_S[m-V_Y[m])\cdot G_\sigma(y_{k+1} - y_{k+1-m})\cdot(y_{k+1} - y_{k+1-m})(\mathbf{X}_{k+1} - \mathbf{X}_{k+1-m}) - \sum_{m=1}^{M}\frac{1}{(N-m+1)\sigma^2}(V_S[m-V_Y[m])\cdot G_\sigma(y_{k+1-N+m} - y_{k+1-N})\cdot(y_{k+1-N+m} - y_{k+1-N})\cdot(\mathbf{X}_{k+1-N+m} - \mathbf{X}_{k+1-N})$$

$$= \left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k}^{S} + \sum_{m=1}^{M}\frac{1}{(N-m+1)\sigma^2}(V_S[m-V_Y[m])\cdot\big[G_\sigma(y_{k+1} - y_{k+1-m})\cdot(y_{k+1} - y_{k+1-m})(\mathbf{X}_{k+1} - \mathbf{X}_{k+1-m}) - G_\sigma(y_{k+1-N+m} - y_{k+1-N})\cdot(y_{k+1-N+m} - y_{k+1-N})\cdot(\mathbf{X}_{k+1-N+m} - \mathbf{X}_{k+1-N})\big] \quad (12)$$

The equation (12) shows that the next gradient can be obtained recursively from the current gradient in the steady state.

Secondly, the relationship of the initial state gradient between the next time gradient and the current gradient is

$$\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k+1}^{I} = \sum_{m=1}^{M}\sum_{i=1}^{k+1}\frac{1}{(k-m+2)\sigma^2}(V_S[m] - V_Y[m])\cdot G_\sigma(y_i - y_{i-m})\cdot(y_i - y_{i-m})(\mathbf{X}_i - \mathbf{X}_{i-m})$$

$$= \sum_{m=1}^{M}\sum_{i=1}^{k}\frac{1}{(k-m+2)\sigma^2}(V_S[m] - V_Y[m])\cdot G_\sigma(y_i - y_{i-m})\cdot(y_i - y_{i-m})(\mathbf{X}_i - \mathbf{X}_{i-m}) + \sum_{m=1}^{M}\frac{1}{(k-m+2)\sigma^2}(V_S[m-V_Y[m])\cdot G_\sigma(y_{k+1} - y_{k+1-m})\cdot(y_{k+1} - y_{k+1-m})(\mathbf{X}_{k+1} - \mathbf{X}_{k+1-m})$$

$$= \sum_{m=1}^{M}\sum_{i=1}^{k}\frac{(k-m+1)}{(k-m+2)(k-m+1)\sigma^2}(V_S[m] - V_Y[m])\cdot G_\sigma(y_i - y_{i-m})\cdot(y_i - y_{i-m})(\mathbf{X}_i - \mathbf{X}_{i-m}) + \sum_{m=1}^{M}\frac{1}{(k-m+2)\sigma^2}(V_S[m-V_Y[m])\cdot G_\sigma(y_{k+1} - y_{k+1-m})\cdot(y_{k+1} - y_{k+1-m})(\mathbf{X}_{k+1} - \mathbf{X}_{k+1-m}) \quad (13)$$

In order for the double summation term in (13) to be equal to (8), $\frac{(k-m+1)}{(k-m+2)}$ should be 1. For small k in the initial state, $(k-m+1)$ does not approach $(k-m+2)$ so that $\left.\frac{\partial P_{CD}}{\partial \mathbf{W}}\right|_{k+1}^{I}$ is not expressed as a recursive form.

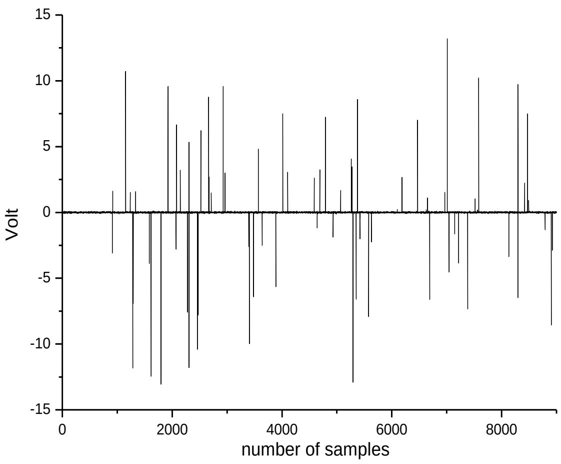

Fig. 2. Impulsive noise for the experiment.

From the analysis presented above and the recursive form (12), we propose that the weight update equation (5) is carried out using (8) in the initial state and the recursive gradient estimation (12) in $(k > N)$. From (12) we can notice that the proposed algorithm reduces the computations of the double summation $(N+1)M - (M+1)M/2$ to the computations of single summation 2*M*. That is, the conventional gradient method of correntropy algorithm has *O*(*NM*), but the proposed method has only *O*(*M*) while keeping the same performance.

IV. Results and Discussion

In this section, it is experimented whether the proposed gradient estimation of the correntropy algorithm yields the same results as the block-processed gradient estimation, and then the MSE learning performance of the proposed algorithm and CMA is compared. The 4 symbol points of $a_i \{-3, -1, 1, 3\}$ are transmitted through the multipath channel composed of 3 paths $h_0 = 0.26$, $h_1 = 0.93$, $h_2 = 0.26$. The constant modulus R_2 is 8.2. The impulsive noise to be added to the channel output is generated according to the work [5] with variance 50 and incident rate 0.01 as depicted in Fig. 2. The variance of the background white noise is 0.001. The equalizer length is L=11 and the number N in the size of the data block is 30. The number of lags M is 20 and the kernel size σ is 2.8 and the convergence parameter μ=0.02 is used. The parameter values are chosen that produce the lowest steady state MSE in this simulation. For MSE performance comparison, we tested the well known CMA, the proposed algorithm. The convergence parameter for the CMA is 0.000001.

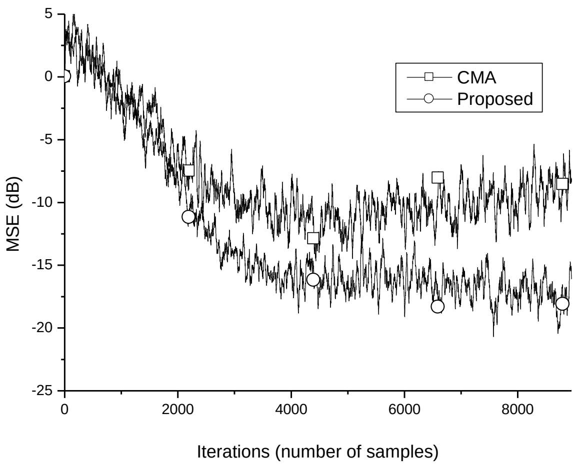

Fig. 3. MSE learning curves.

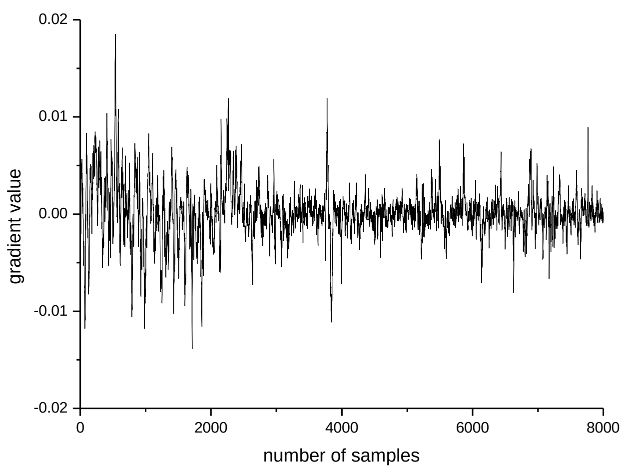

Fig. 4. Gradient estimation results of the conventional block-processing method and the proposed recursive method.

In the Fig. 3 we observe that the proposed algorithm converges but the CMA fails to converge revealing its vulnerability to impulsive noise. In the noise of the strong and frequent impulses as shown in Fig. 2, the proposed algorithm reaches a significantly low steady state MSE around -17 dB while the CMA diverges.

Fig. 4 shows the trace of gradient estimation results for the first tap weight (the other tap weights are omitted in this paper just for the page-limit). The dotted line is the result of the conventional block-processing method by (6) and the solid line is for the proposed estimation method. We find that the two gradient methods yield exactly the same estimation results proving justification of the proposed estimation.

These results indicate that the heavy computational complexity of the conventional correntropy algorithm that may be inappropriate for implementation can be significantly reduced by the proposed method with its performance being preserved.

V. Conclusion

Wireless links in indoor sensor networks have distortions due to multipath fading from reflections and impulsive noise from indoor electric devices. In these harsh environments, blind correntropy equalization algorithm yields superior MSE performance compared with the well known CMA.

A main drawback of the correntropy algorithm is a heavy computational complexity induced from some double summation operations at each iteration time. In this paper, a recursive gradient estimation for weight updates of the correntropy algorithm has been proposed in order to reduce its computational burden. It is analyzed that the proposed method reduces the computations $(N+1)M - (M+1)M/2$ from the double summation to $2M$ from two single summations of the proposed method. The simulation results show that the conventional and proposed gradient estimation methods yield exactly the same estimation traces proving justification of the proposed estimation. These results lead us to the conclusion that the proposed method having a significantly reduced computational complexity can be used in the implementation of reliable and efficient indoor wireless sensor networks.

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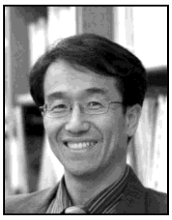

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