On the Secure Outage Performance for Dual-Hop Network Coding Systems with Cooperative Jamming

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Abstract—In this paper, a design of cooperative network coding (NC) and jammer relay (JR) is proposed for this two-way network in order to improve the security of the data exchange. Considering the system over Rayleigh fading channels in the presence of one eavesdropper and two relays, we study secrecy outage (including the probability of non-zero secrecy capacity and secure outage probability), respectively. In this paper, one relay is regarded as NC relay which is used to make codes by xor in order to prevent eavesdropper’s interception while the other one is regarded as a jammer relay which can send interference to eavesdropper. We derive exact expressions for the secure outage probability. The accuracy of our performance analysis is verified by simulation results.

Index Terms—Network coding, jammer relay, secure outage probability, secure communications.

I. INTRODUCTION

The broadcast nature of the wireless medium makes the communication process vulnerable to eavesdroppers which are in the coverage area of the transmission. Thus, security in physical layer of wireless communication networks has taken on an increasingly important role. Traditionally, security was viewed as an independent issue addressed above the physical layer until eavesdropping attack was first studied in the 1970s by Wyner [1] and later by Csiszár and Korner [2].

Recently, there has been a considerable recent attention on studying physical layer secure in cooperative communication scenarios. The main idea is to exploit user cooperation in facilitating the transmission of confidential messages from the source to the destination. In [3], conventional method is using a jammer relay to send jamming to eavesdroppers in order to interfere its capture ability. [4] adopts two types of relays, one is used to transmit confidential messages and the other one is used to send interference to the eavesdropper. To be different from [4], [5] and [6] use the destination node as the jammer which sends interference to the untrusted relay (potential eavesdropper). In [7], destination sends intended jamming noise to relay and then relay transmit messages with noise to destination and eavesdropper, in another word, this relay is both a transfer station and a jammer. In [8], the authors do not consider the direct link between eavesdropper and source while the destination sends an intended jamming noise to the relay, referred to as cooperative jamming.

In this work, we consider network coding (NC) as the cooperation scheme as it is much more superior to other relaying schemes in secure communication. NC disguises messages by the way that NC relay takes several packets and combines them together for transmission. Thus, it can improve a network’s throughput, efficiency and scalability, as well as resilience to attacks and eavesdropping [9]. In this paper, we study the secure outage performance for a dual-hop NC cooperative system in presence of an eavesdropper and two relays. The closed-form expressions of the probability of nonzero secrecy capacity and secure outage probability have been derived.

II. SYSTEM MODEL

Consider a two-way network consisting of two relays, two sources, one destination and one eavesdropper, as shown in Fig. 1. All nodes in the network are equipped with a single half-duplex antenna. In phase 1, both $S_1$ and $S_2$ send their message $m_{S_1}$ and $m_{S_2}$ to the relay node $R_1$ and $S_1$ also sends its message to the destination $D$. The received signals at $R_1$ can be given by

\[ y_{S_1,R_1} = \sqrt{P_S} h_{S_1,R_1} m_{S_1} + n_{R_1}, \quad (1) \]

and

\[ y_{S_2,R_1} = \sqrt{P_S} h_{S_2,R_1} m_{S_2} + n_{R_1}, \quad (2) \]

and the received signal at $D$ can be given by

\[ y_{S_1,D} = \sqrt{P_S} h_{S_1,D} m_{S_1} + n_D, \quad (3) \]
Pr\{C ≤ R\} = 1 - \left\{\exp\left[-\left(\alpha_{S,R} + \alpha_{S,O} + \alpha_{R,E}\right)(2^{2R} - 1)\right]\right\}

\[ \frac{\alpha_{S,E}}{\alpha_{S,E}} + \alpha_{S,R} 2^{2R} \exp\left(\frac{\alpha_{S,R} \alpha_{S,E} 2^{2R}}{\alpha_{S,E}}\right) + E\left(\frac{-\alpha_{S,R} \alpha_{S,E} 2^{2R}}{\alpha_{S,E}}\right) \alpha_{S,E} \exp\left(-\alpha_{S,R} 2^{2R} + \alpha_{S,O}\right) \]

(11)

where \(P_i\) is the transmit power at node \(i\) (\(i \in \{S_1, S_2, R_1\}\)); \(n_j\) (\(j \in \{R_1, D\}\)) is the complex additive white Gaussian noise (AWGN) with zero mean and variance \(\sigma^2\) and \(h_{ij}\) is the channel coefficient from the node \(i\) to node \(j\).

In phase 2, \(R_1\) decodes out the message \(m_{S_1}\) and \(m_{S_2}\) from the received signal \(y_{S_1}\) and \(y_{S_2}\) and then sends the signal \(m_{R_1} = m_{S_1} \oplus m_{S_2}\) to node \(D\). The received signal \(y_{R,D}\) at \(D\) is given by

\[ y_{R,D} = \sqrt{P_{R_1} h_{R,D}} m_{R_1} + n_D. \] (4)

At the node \(D\), \(D\) can decode out the message \(m_{S_1}\) by the expression \(m_{S_1} = m_{R_1} \oplus m_{R_2}\), so \(D\) can get both \(m_{S_1}\) and \(m_{S_2}\).

In both phase 1 and 2, the eavesdropper node \(E\) can capture partial messages of the link from \(S_i\) to \(R_i\) and the link from \(R_1\) to \(D\), at the meantime, the node \(R_2\) sends interference to \(E\).

### III. Secure Outage Probability

In this section, we derive the closed-form expressions for PNSC and SOP.

Let \(\gamma_{ij}\) represents the Signal-to-Noise-Ratio (SNR) of the link \(i\)-to-\(j\) (\(i, j \in \{S_1, S_2, D, R_1, E\}\), we have

\[ \gamma_{ij} = P_i h_{ij} / N_0, \] (5)

where \(N_0\) is the power of the AGWN.

For \(h_{ij}\) follows Rayleigh distribution, \(\gamma_{ij}\) follows exponential distribution, \(\gamma_i \sim \exp\left(\frac{N_0}{2\sigma_i^2 P_i}\right)\).

The secrecy capacity of the link from \(S_i\) to \(R_i\) can be given by

\[ C_i = C_{S,R} = \frac{1}{2} \log_2 \left(1 + \gamma_{S,R}\right), \] (6)

so we can get the following secrecy capacities which can be given by

\[ C_2 = C_{S,R} = \frac{1}{2} \log_2 \left(1 + \gamma_{S,R}\right) - \frac{1}{2} \log_2 \left(1 + \gamma_{S,E}\right), \] (7)

\[ C_3 = C_{S,O} = \frac{1}{2} \log_2 \left(1 + \gamma_{S,O}\right), \] (8)

\[ C_4 = C_{R,E} = \frac{1}{2} \log_2 \left(1 + \gamma_{R,E}\right). \] (9)

Then the PNSC can be expressed as

\[ \Pr\{C > 0\} = \Pr\{\min\{C_1, C_2, C_3, C_4\} > 0\} \]

\[ = 1 - \frac{\alpha_{S,R} \alpha_{S,E}}{\alpha_{S,E}} \exp\left(\frac{\alpha_{S,R} \alpha_{S,E} 2^{2R}}{\alpha_{S,E}}\right) + E\left(\frac{-\alpha_{S,R} \alpha_{S,E} 2^{2R}}{\alpha_{S,E}}\right) \alpha_{S,E} \exp\left(-\alpha_{S,R} 2^{2R} + \alpha_{S,O}\right) \] (10)

where \(\alpha_i = \frac{N_0}{2\sigma_i^2 P_i}\) and \(E(x)\) is exponential integral function. The detailed derivation of Eq.(10) can be found in Appendix.

And the secure outage probability (SOP) can be given by the Eq. (11), where \(\alpha_i = \frac{N_0}{2\sigma_i^2 P_i}\) and and \(E(x)\) is exponential integral function. The detailed derivation of Eq.(11) can be found in Appendix.

### IV. Numerical Results and Discussion

In this section, we present Monte Carlo simulation, theoretical and analytical results corresponding to the secrecy outage (including the PNSC and SOP) over independent Rayleigh fading channels. The main parameters used in simulation and analysis are set as \(\sigma_{sd} = 1.5\), \(\sigma_{re} = 2.5\), \(\sigma_\omega = \sqrt{3/2}\), \(\sigma_\omega = \sqrt{3/2}\), \(P_s = P_0 = P_h = P_{R_1} = 1\), \(\lambda = \sigma_{S,R}/\sigma_{R,E}\).

In Fig. 2-Fig. 3, we compare simulation, theoretical and analytical results of PNSC. It is seen that the simulation results match very well with the theoretical and analytical results. Further, it could be observed that PNSC for a higher \(\omega\) outperforms that of a lower \(\omega\). Fig. 2 describes that the PNSC versus \(\omega\). We also find that PNSC has been improved while \(\omega\) increases because a larger \(\omega\) means a better S-R1 link. Fig. 3 describes that PNSC versus SNR. We can see that the increasing of SNR can improve PNSC.
In Fig. 4 - Fig. 5, we present the comparisons among simulation, theoretical and analytical results of SOP. It is obvious that simulation, theoretical and analytical results match each other very well. Further, we also see that SOP for a higher \( \sigma_{r_2} \) outperforms that of a lower \( \sigma_{r_2} \). Some similar observations can be found in Figs. 4 and 5 with the ones found in Fig. 2 and Fig. 3, i.e., the effect of \( \omega \) and SNR on PNSC and SOP.

V. CONCLUSION

In this paper, we have investigated the secure outage performance for cooperative NC communication systems over independent Rayleigh fading channels. The closed-form expressions for the probability of nonzero secrecy capacity and secure outage probability have been derived. The simulation results are in excellent agreement with the theoretical and analytical results obtained in this work.

APPENDIX

In order to get the PNSC and SOP, we can first calculate the probabilities of that each instantaneous secrecy capacity \( C_i (i \in \{1,2,3,4\}) \) is below a target secrecy rate \( R \).

We have already known that \( \gamma_{i,1} \) follows an exponential distribution, so the SOP of the \( C_i (i \in \{1,3,4\}) \) can be given by

\[
\Pr\{C_i \leq R\} = 1 - \exp\left[-\alpha_{s,x} \left(2^{R-1}\right)\right].
\]  
(12)

\[
\Pr\{C_i \leq R\} = 1 - \exp\left[-\alpha_{r,x} \left(2^{R-1}\right)\right].
\]  
(13)

\[
\Pr\{C_i \leq R\} = 1 - \exp\left[-\alpha_{r,d} \left(2^{R-1}\right)\right].
\]  
(14)

If \( R = 0 \), we can get

\[
\Pr\{C_i \leq 0\} = \Pr\{\gamma_{S,R} \leq 0\} = 0,
\]  
(15)

\[
\Pr\{C_i \leq 0\} = \Pr\{\gamma_{S,D} \leq 0\} = 0,
\]  
(16)

\[
\Pr\{C_i \leq 0\} = \Pr\{\gamma_{R,D} \leq 0\} = 0.
\]  
(17)

The item \( \Pr\{C_2 \leq R\} \) can be given by

\[
\Pr\{C_2 \leq R\} = \Pr\left[\frac{1 + \gamma_{s,b} \leq 2^{R}}{1 + \gamma_{s,e}}\right] = \Pr\{\gamma_{s,b} \leq 2^{R} (1+\gamma_{e})^{-1}\} = \int_{0}^{\gamma_{s,b}} \Pr\{\gamma_{s,b} \leq 2^{R} (1+s)^{-1}\} f_{\gamma_{s,b}}(s) ds
\]  
(18)

where \( F_{\gamma_{s,b}}(s) \) denotes the probability distribution function of \( \gamma_{s} \) which denotes \( \gamma_{s,e} \) and \( f_{\gamma_{s,b}}(s) \) denotes the probability density function of \( \gamma_{s} \). We use \( f_{x}(x) \) and \( f_{i}(x) \) respectively denote the PDF of \( \gamma_{S,i} \) and \( \gamma_{R,e} \), so the calculation of \( \gamma_{s} \) can be given by

\[
f_{\gamma_{s,b}}(s) = \int_{0}^{\infty} x f_{\gamma_{s,b}}(sx)f_{\gamma_{s,b}}(x) dx = \int_{0}^{\infty} x \alpha_{s,e} \exp\left(-\alpha_{s,e} sx\right) \alpha_{s,e} \exp\left(-\alpha_{s,e} x\right) dx = \alpha_{s,e} \alpha_{s,b} \int_{0}^{\infty} x \exp\left(-\left(\alpha_{s,e} s + \alpha_{s,e}\right) x\right) dx
\]
and the SOP can be given by

\[ \Pr \{C_2 \leq R \} = \Pr \left[ \frac{1 + \gamma_{2, R}}{2} \leq \frac{1}{1 + \gamma_{2, R}} \right] 
= \int_0^1 \Pr \left[ \gamma_{2, R} \leq \frac{1}{1 + \gamma_{2, R}} \right] f_{\gamma_{2, R}}(s) \, ds 
= \int_0^1 \left( 1 - \exp \left[ -\alpha_{2, R} \left( 1 + s \right) \right] \right) \frac{\alpha_{2, R} \alpha_{E}}{\alpha_{2, E} + \alpha_{E}} \, ds 
= 1 - \exp \left[ -\frac{\alpha_{2, R} \alpha_{E}}{\alpha_{2, E}} \exp \left( \frac{\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right) E_i \left( \frac{-\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right) \right]. \tag{20} \]

When \( R = 0 \), we can get

\[ \Pr \{C_1 \leq 0\} = \Pr \left[ \frac{1}{1 + \gamma_{2, R}} \leq 1 \right] = \Pr \left[ \gamma_{2, R} \leq \gamma_{1} \right] 
= \frac{\alpha_{2, R} \alpha_{E}}{\alpha_{2, E}} \exp \left( \frac{\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right) E_i \left( \frac{-\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right). \tag{21} \]

Finally, the PNAC can be given by

\[ \Pr \{C > 0\} = \Pr \left[ \min \{C_1, C_2, C_3, C_4\} > 0 \right] 
= \Pr \{C_1 > 0\} \Pr \{C_2 > 0\} \Pr \{C_3 > 0\} \Pr \{C_4 > 0\} 
= \prod_{i=1}^{4} (1 - \Pr \{C_i \leq 0\}) 
= 1 + \frac{\alpha_{2, R} \alpha_{E}}{\alpha_{2, E}} \exp \left( \frac{\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right) E_i \left( \frac{-\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right), \tag{22} \]

and the SOP can be given by

\[ \Pr \{C \leq R\} = \Pr \left[ \min \{C_1, C_2, C_3, C_4\} \leq R \right] 
= 1 - \Pr \left[ \min \{C_1, C_2, C_3, C_4\} > R \right] 
= 1 - \prod_{i=1}^{4} (1 - \Pr \{C_i \leq R\}) 
= 1 - \exp \left[ -\alpha_{2, R} \left( 2^{2^R} - 1 \right) \right] \exp \left[ -\alpha_{2, R} \left( 2^{2^R} - 1 \right) \right] 
\left[ \frac{\alpha_{2, R} \alpha_{E}}{\alpha_{2, E}} \exp \left( \frac{\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right) E_i \left( \frac{-\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right) \right] 
\left[ \frac{\alpha_{2, R} \alpha_{E}}{\alpha_{2, E}} \exp \left( \frac{\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right) E_i \left( \frac{-\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right) \right] 
\left[ \frac{\alpha_{2, R} \alpha_{E}}{\alpha_{2, E}} \exp \left( \frac{\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right) E_i \left( \frac{-\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right) \right] 
\left[ \frac{\alpha_{2, R} \alpha_{E}}{\alpha_{2, E}} \exp \left( \frac{\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right) E_i \left( \frac{-\alpha_{2, R} \alpha_{E} 2^{2^s}}{\alpha_{2, E}} \right) \right] \tag{23} \]
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