Generalized Function Projective Lag Synchronization in Fractional-Order Chaotic Systems

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***Abstract*—In this paper, a new synchronization of fractional-order chaotic systems called generalized function projective lag synchronization is introduced. This synchronization method is a generalization of function projective synchronization and function projective lag synchronization. Based on the stability theorem of linear fractional order systems, a suitable nonlinear fractional-order controller is designed for the synchronization of different structural systems. Three examples are given to verify the effectiveness of the proposed method.**

***Index Terms*—Chaos, fractional order, generalized function projective lag synchronization, nonlinear controller.**

I. Introduction

Since pioneering and meaningful work of Pecora and Carroll [1], chaotic synchronization has become a hot topic and been studied by more and more scholars in recent decades. It has many applications in the field of physics, chemistry, biology, and others especially in secure communication. From the existing literature, there are many different types of synchronization method, such as complete synchronization [2]-[5], projective synchronization [6], generalized synchronization [7], [8], robust synchronization [9], function projective synchronization (FPS) [10], function projective lag synchronization (FPLS) [11]. There are many control scheme utilizing in chaotic synchronization, such as adaptive control [12]-[14], sliding mode control [15]-[17], fuzzy sliding mode control [18], [19]. In [20], a reference system is added to make the synchronization more complex. Inspired by [20], we introduce a new type of synchronization method named generalized function projective lag synchronization (GFPLS) which is a generalization of FPS and FPLS.

The organization of this paper is as follows. The GFPLS method is introduced in Section II. Simulation and results are presented in Section III. In Section IV, conclusions are proposed.

II. The GFPLS Method

A. The Definition of GFPLS

Choose fractional-order drive system as:

$$\begin{aligned} D^{\varepsilon}x(t) &= l(x(t)) \\ x(t) &= x(0), t \in [-\omega, 0] \end{aligned} \tag{1}$$

where D^{ε} signifies the fractional-order differential operator. $\varepsilon \in (0,1)$ denotes the fractional order of drive system, $l: R^c \to R^c$ indicates the continuous function of drive system. $x(t) = (x_1(t), x_2(t), \cdots x_c(t))^T \in R^c$ indicates the state vector of drive system, c denotes the dimension of drive system. $\omega > 0$ is the time delay. The fractional-order response system is defined as:

$$D^{\eta}y(t) = f(y(t)) + u(x(t-\omega), y(t), z(t)) \tag{2}$$

where D^{η} signifies the fractional-order differential operator. $\eta \in (0,1)$ denotes the fractional order of response system, $f: R^d \to R^d$ indicates the continuous function. $y(t) = (y_1(t), y_2(t), \cdots y_d(t))^T \in R^d$ indicates the state vector of response system, d denotes the dimension of response system. $u(t) = (u_1(t), u_2(t), \cdots u_d(t))^T \in R^d$ is the nonlinear controller to be designed later. Choose fractional- order reference system as:

$$D^{q}z(t) = h(z(t)) \tag{3}$$

where D^{q} signifies the fractional-order differential operator. $q \in (0,1)$ indicates the fractional order of reference system, $h: R^d \to R^d$ denotes the continuous function. $z(t) = (z_1(t), z_2(t), \cdots z_d(t))^T \in R^d$ denotes the state vector, System (3) is an attractor. Define error state vector as:

$$e(t) = y(t) - C(x(t-\omega))x(t-\omega) - z(t) \tag{4}$$

where $C(x(t-\omega)) \in R^{d\times c}$, $x_{\omega} = x(t-\omega)$.

There exists two conditions, one condition is system dimension $d \le c$

$$C(x_{\omega}) = \begin{pmatrix} K(x_{\omega}) & 0 \\ 0 & Q(x_{\omega}) \end{pmatrix}$$

where the matrix

$$K(x_{\omega}) = diag\{c_1(x_{\omega}), c_2(x_{\omega}), \cdots, c_{\tau-1}(x_{\omega})\},$$

$$\tau = \min\{c, d\} \text{ and } Q(x_{\omega}) = (c_{\tau}(x_{\omega}), \cdots, c_{\mu}(x_{\omega})),$$

$\mu = \max\{c, d\}$, the other condition is system dimension $d > c$, the matrix is defined as

$$C(x_{\omega}) = \begin{pmatrix} K(x_{\omega}) & 0 \\ 0 & Q^T(x_{\omega}) \end{pmatrix}$$

Definition 1: GFPLS is realized. Assume there exists a nonlinear controller $u(x_{\omega}, y(t), z(t))$ such that $\lim_{t\to\infty} \|e(t)\| = 0$.

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Remark 1: When system (3) denotes constant zero, GFPLS degenerated to FPLS.

Remark 2: When system (3) denotes constant zero and time delay is zero, GFPLS degenerated to FPS.

Remark 3: System (3) can be other attractors, such as chaos, hyperchaos, periodic function, and quasi period function.

B. The Stability Analysis of GFPLS

Consider controller as:

$$u = D^{\eta}(C(x_\omega)x_\omega) + D^{\eta}(z) - f(C(x_\omega)x_\omega) - f(z) + v + G(x_\omega, y(t), z(t))e \quad (5)$$

where $v \in R^d$ is the compensation vector, $G(x_\omega, y(t), z(t)) \in R^{d\times d}$ is a polynomial matrix. According to (2), (3) and (5), we obtain the fractional-order error system as follows:

$$D^{\eta}e(t) = (F(x_\omega, y(t), z(t)) + G(x_\omega, y(t), z(t)))e(t) \quad (6)$$

where $F(x_\omega, y(t), z(t))e(t) = f(y(t)) - f(C(x_\omega)x_\omega) - f(z(t)) + v$.

Proposition 1: $C(x_\omega)$ and time delay is given, GFPLS is accomplished if there exists $G(x_\omega, y(t), z(t)) \in R^{d\times d}$ such that

$$F(x_\omega, y(t), z(t)) + G(x_\omega, y(t), z(t)) = -N(x_\omega, y(t), z(t)) \quad (7)$$

where $N = [n_{i,j}] \in R^{d\times d}$, $i \neq j, n_{i,j} = -n_{j,i}$, $i{=}j, n_{i,j} \in R^{+}$

Proof: Assume λ is an arbitrary eigenvalue of the matrix $F(x_\omega, y(t), z(t)) + G(x_\omega, y(t), z(t))$ and the related nonzero eigenvector denotes ζ. We may get

$$(F(x_\omega, y(t), z(t)) + G(x_\omega, y(t), z(t)))\zeta = \lambda\zeta \quad (8)$$

Multiplying the ζ^H at the left of (8), we have

$$\zeta^H(F(x_\omega, y(t), z(t)) + G(x_\omega, y(t), z(t)))\zeta = \lambda\zeta^H\zeta \quad (9)$$

where H denotes conjugate transpose of a matrix, λ^* also indicates an eigenvalue of the matrix $F(x_\omega, y(t), z(t)) + G(x_\omega, y(t), z(t))$

Similarly, we have

$$\zeta^H(F(x_\omega, y(t), z(t)) + G(x_\omega, y(t), z(t)))^H\zeta = \lambda^*\zeta^H\zeta \quad (10)$$

According to (9) and (10), we obtain

$$\begin{aligned}\lambda + \lambda^* &= \zeta^H[(F(x_\omega, y, z) + G(x_\omega, y, z)) + \\ &\quad (F(x_\omega, y, z) + G(x_\omega, y, z))^H]\zeta / \zeta^H\zeta \\ &= -\zeta^H\psi\zeta / \zeta^H\zeta\end{aligned} \quad (11)$$

where $\zeta^H\zeta > 0$, $\psi = (N(x_\omega, y, z) + N(x_\omega, y, z)^H) \in R^{d\times d}$.

We are able to obtain that ψ denotes a real positive-definite diagonal matrix. Thus, $\zeta^H\psi\zeta > 0$, we can have

$$\lambda + \lambda^* = 2real(\lambda) < 0 \quad (12)$$

Hence, we can obtain

$$|\arg(\lambda)| > \pi/2 > \eta\pi/2 \quad (13)$$

Based on the fractional-order stability theorem proposed in [21], the error system (6) asymptotically stabilizes at origin. GFPLS is achieved. The proof is completed.

III. Simulation and Results

In this section, three examples presented in [11] are utilized to demonstrate the effectiveness of the GFPLS with the same and different dimension. A predictor-corrector method proposed in [22], [23] is used to solve fractional-order differential equations.

A. GFPLS with Same Dimension d=c

Choose the fractional-order Rössler system as drive system.

$$\begin{aligned}D^{\varepsilon}x_1(t) &= -x_2(t) - x_3(t) \\ D^{\varepsilon}x_2(t) &= x_1(t) + \alpha_1 x_2(t) \\ D^{\varepsilon}x_3(t) &= \alpha_2 + x_1(t)x_3(t) - \alpha_3 x_3(t) \\ x(t) &= x(0), t \in [-\omega, 0]\end{aligned} \quad (14)$$

When $\varepsilon{=}0.95, (\alpha_1, \alpha_2, \alpha_3) = (0.4, 0.2, 10)$, $x(0) = (0.5, 0.5, 0.5)^T$, drive system shows chaotic attractor drawn in Fig. 1.

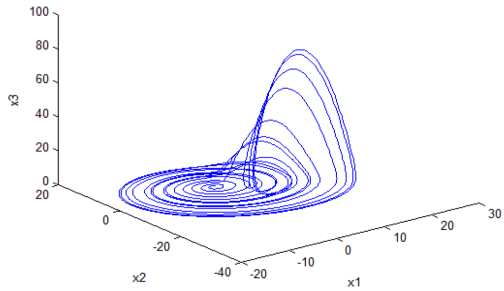

Fig. 1. The phase trajectory of the fractional-order Rössler system.

Choose the fractional-order Lü system as response system.

$$\begin{aligned}D^{\eta}y_1(t) &= \beta_1(y_2(t) - y_1(t)) + u_1(x_\omega, y(t), z(t)) \\ D^{\eta}y_2(t) &= -y_1(t)y_3(t) + \beta_2 y_2(t) + u_2(x_\omega, y(t), z(t)) \\ D^{\eta}y_3(t) &= y_1(t)y_2(t) - \beta_3 y_3(t) + u_3(x_\omega, y(t), z(t))\end{aligned} \quad (15)$$

When $\eta{=}0.9, (\beta_1, \beta_2, \beta_3) = (35, 28, 3)$, $y(0) = (4.2, 3.5, 11)^T$, response system denotes chaotic attractor shown in Fig. 2.

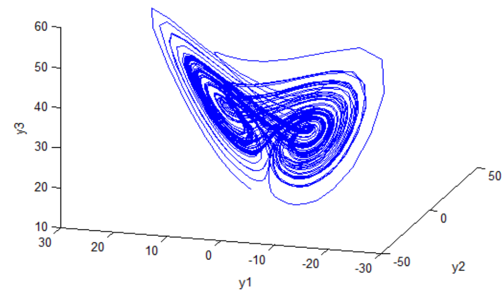

Fig. 2. The phase trajectory of the fractional-order Lü system.

Choose the fractional-order Chen system as reference system.

$$\begin{aligned}D^{q}z_1(t) &= \gamma_1(z_2(t) - z_1(t)) \\ D^{q}z_2(t) &= (\gamma_3 - \gamma_1)z_1(t) - z_1(t)z_3(t) + \gamma_3 z_2(t) \\ D^{q}z_3(t) &= z_1(t)z_2(t) - \gamma_2 z_3(t)\end{aligned} \quad (16)$$

When $q = 0.95, (\gamma_1, \gamma_2, \gamma_3) = (35, 3, 28)$, $z(0) = (15, 12, 31)^T$, reference system indicates chaotic attractor drawn in Fig. 3.

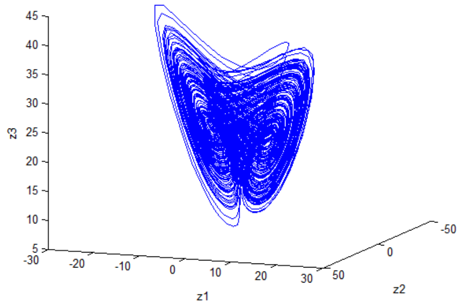

Fig. 3. The phase trajectory of the fractional-order Chen system.

The error states are defined as

$$e_i(t) = y_i(t) - c_i(x_\omega)x_{i\omega} - z_i(t), x_{i\omega} = x_i(t-\omega),$$

$i = 1,2,3$. According to (5) and (15), we can get

$$F(x_\omega, y(t), z(t)) = \begin{pmatrix} -\beta_1 & \beta_1 & 0 \\ -(c_3(x_\omega)x_{3\omega} + z_3(t)) & \beta_2 & -y_1(t) \\ y_2(t) & (c_1(x_\omega)x_{1\omega} + z_1(t)) & -\beta_3 \end{pmatrix}$$

$$G(x_\omega, y(t), z(t)) = \begin{pmatrix} 0 & (c_3(x_\omega)x_{3\omega} + z_3(t) - \beta_1) & -y_2(t) \\ 0 & -\beta_2 - d_1 & y_1(t) \\ 0 & -(c_1(x_\omega)x_{1\omega} + z_1(t)) & 0 \end{pmatrix}$$

where $d_1 > 0$, we can obtain

$$F(x_\omega, y(t), z(t)) + G(x_\omega, y(t), z(t)) = -N(x_\omega, y(t), z(t))$$

where $N = [n_{i,j}] \in R^{d \times d}$, $i \neq j, n_{i,j} = -n_{j,i}$, $i{=}j, n_{i,j} \in R^+$, $i, j = 1,2,3,$.

The GFPLS with same dimension is realized based on proposition 1.

When $C(x_\omega) = diag\{10x_{2\omega} + 35, x_{3\omega} - 12.5, 4x_{3\omega} - 20x_{1\omega}\}$, $\omega = 0.5, d_1 = 5$, the error state curves of GFPLS with same dimension are shown in Fig. 4.

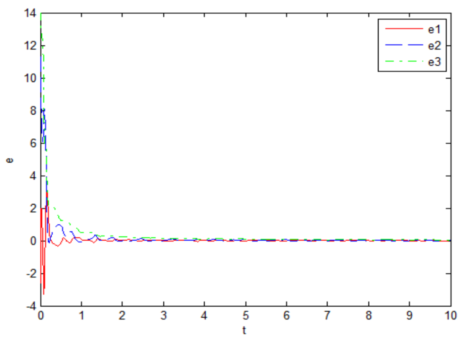

Fig. 4. The error state curves of GFPLS with same dimension.

B. GFPLS with Different Dimension d<c

Choose the fractional-order hyperchaotic Lorenz system as drive system

$$\begin{aligned} D^\varepsilon x_1(t) &= \alpha_1(x_2(t) - x_1(t)) + x_4(t) \\ D^\varepsilon x_2(t) &= \alpha_2 x_1(t) - x_2(t) - x_1(t)x_3(t) \\ D^\varepsilon x_3(t) &= x_1(t)x_2(t) - \alpha_3 x_3(t) \\ D^\varepsilon x_4(t) &= -x_2(t)x_3(t) - \alpha_4 x_4(t) \\ x(t) &= x(0), t \in [-\omega, 0] \end{aligned} \tag{17}$$

When $\varepsilon = 0.98, (\alpha_1, \alpha_2, \alpha_3, \alpha_4) = (10, 28, 8/3, 1)$, $x(0) = (-2.2, -6, 8.3, -9)^T$, drive system denotes hyperchaotic attractor shown in Fig. 5.

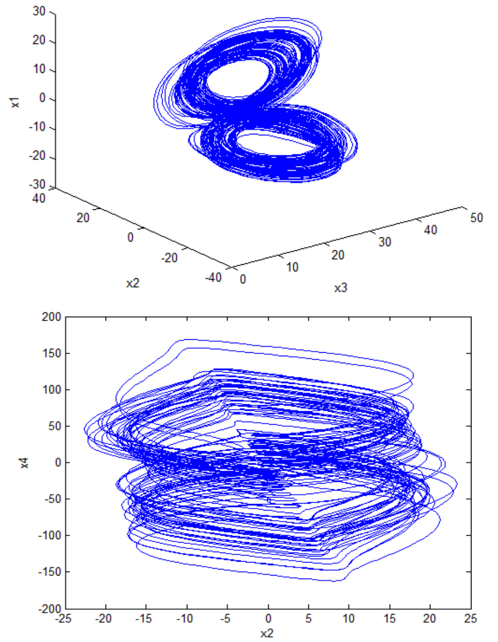

Fig. 5. The phase trajectory of the fractional-order hyperchaotic Lorenz system.

Choose the fractional-order Rössler system as response system.

$$\begin{aligned} D^\eta y_1(t) &= -y_2(t) - y_3(t) + u_1(x_\omega, y(t), z(t)) \\ D^\eta y_2(t) &= y_1(t) + \beta_1 y_2(t) + u_2(x_\omega, y(t), z(t)) \\ D^\eta y_3(t) &= \beta_2 + y_1(t)y_3(t) - \beta_3 y_3(t) + u_3(x_\omega, y(t), z(t)) \end{aligned} \tag{18}$$

Choose the fractional-order Lü system as reference system.

$$\begin{aligned} D^q z_1(t) &= \gamma_1(z_2(t) - z_1(t)) \\ D^q z_2(t) &= -z_1(t)z_3(t) + \gamma_2 z_2(t) \\ D^q z_3(t) &= z_1(t)z_2(t) - \gamma_3 z_3(t) \end{aligned} \tag{19}$$

Based on (4), we can obtain the error states

$$\begin{aligned} e_1(t) &= y_1(t) - c_1(x_\omega)x_{1\omega} - z_1(t) \\ e_2(t) &= y_2(t) - c_2(x_\omega)x_{2\omega} - z_2(t) \\ e_3(t) &= y_3(t) - c_3(x_\omega)x_{3\omega} - c_4(x_\omega)x_{4\omega} - z_3(t) \end{aligned}$$

where $x_{i\omega} = x_i(t-\omega)$, $i = 1,2,3,4$. According to (5) and (18), we can have

$$F(x_\omega, y(t), z(t)) = \begin{pmatrix} 0 & -1 & -1 \\ 1 & \beta_1 & 0 \\ c_3(x_\omega)x_{3\omega} + c_4(x_\omega)x_{4\omega} + z_3(t) & 0 & y_1(t) - \beta_3 \end{pmatrix}$$

$$G(x_\omega, y(t), z(t)) = \begin{pmatrix} -d_2 & 0 & L \\ 0 & -\beta_1 - d_3 & -y_2(t) \\ 0 & y_2(t) & -y_1(t) + \beta_3 - d_4 \end{pmatrix}$$

where $L{=}1 - c_3(x_\omega)x_{3\omega} - c_4(x_\omega)x_{4\omega} - z_3(t)$, $d_2 > 0, d_3 > 0, d_4 > 0$, we can obtain

$$F(x_\omega, y(t), z(t)) + G(x_\omega, y(t), z(t)) = -N(x_\omega, y(t), z(t))$$

where $N = [n_{i,j}] \in R^{d \times d}$, $i \neq j, n_{i,j} = -n_{j,i}$, $i{=}j, n_{i,j} \in R^+$, $i, j = 1,2,3$.

Hence, the GFPLS with different dimension (d<c) is achieved based on proposition 1.

When $\omega = 0.2, d_2 = 1, d_3 = 6, d_4 = 3$,

$$C(x_\omega)=\begin{pmatrix} -x_{2\omega}/2 & & & \\ & x_{2\omega}+x_{3\omega} & & \\ & & 1 & x_{1\omega}/2 \end{pmatrix}.$$

The error state curves of GFPLS with different dimension (d<c) are shown in Fig. 6.

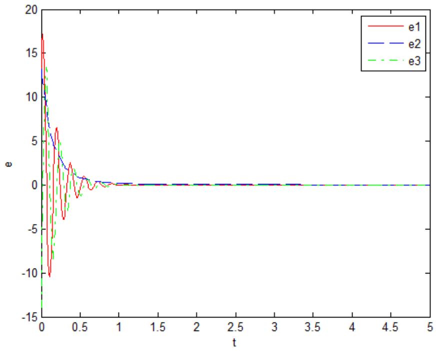

Fig. 6. The error state curves of GFPLS with different dimension (d<c).

C. GFPLS with Different Dimension d>c

Choose the fractional-order Lü system as drive system.

$$\begin{aligned} D^{\varepsilon}x_1(t) &= \alpha_1(x_2(t)-x_1(t)) \\ D^{\varepsilon}x_2(t) &= -x_1(t)x_3(t)+\alpha_2 x_2(t) \\ D^{\varepsilon}x_3(t) &= x_1(t)x_2(t)-\alpha_3 x_3(t) \\ x(t) &= x(0), t\in[-\omega,0] \end{aligned} \tag{20}$$

Choose the fractional-order hyperchaotic Chen system as response system.

$$\begin{aligned} D^{\eta}y_1(t) &= \beta_1(y_2(t)-y_1(t))+y_4(t)+u_1(x_\omega,y(t),z(t)) \\ D^{\eta}y_2(t) &= \beta_2 y_1(t)-y_1(t)y_3(t)+\beta_3 y_2(t)+u_2(x_\omega,y(t),z(t)) \\ D^{\eta}y_3(t) &= y_1(t)y_2(t)-\beta_4 y_3(t)+u_3(x_\omega,y(t),z(t)) \\ D^{\eta}y_4(t) &= y_2(t)y_3(t)+\beta_5 y_4(t)+u_4(x_\omega,y(t),z(t)) \end{aligned} \tag{21}$$

When $\eta=0.96,(\beta_1,\beta_2,\beta_3,\beta_4,\beta_5)=(35,7,12,3,0.5)$, $y(0)=(1.2,2.1,3.1,0.1)^T$, response system indicates the hyperchaotic attractor shown in Fig. 7.

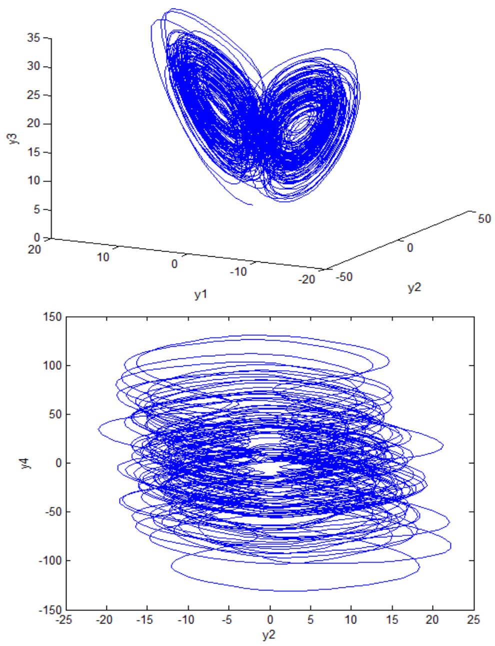

Fig. 7. The phase trajectory of the fractional-order hyperchaotic Chen system.

Choose the fractional-order hyperchaotic Lorenz system as reference system

$$\begin{aligned} D^{q}z_1(t) &= \gamma_1(z_2(t)-z_1(t))+z_4(t) \\ D^{q}z_2(t) &= \gamma_2 z_1(t)-z_2(t)-z_1(t)z_3(t) \\ D^{q}z_3(t) &= z_1(t)z_2(t)-\gamma_3 z_3(t) \\ D^{q}z_4(t) &= -z_2(t)z_3(t)-\gamma_4 z_4(t) \end{aligned} \tag{22}$$

According to (4), we can have the error states

$$\begin{aligned} e_1(t) &= y_1(t)-c_1(x_\omega)x_{1\omega}-z_1(t) \\ e_2(t) &= y_2(t)-c_2(x_\omega)x_{2\omega}-z_2(t) \\ e_3(t) &= y_3(t)-c_3(x_\omega)x_{3\omega}-z_3(t) \\ e_4(t) &= y_4(t)-c_4(x_\omega)x_{3\omega}-z_4(t) \end{aligned}$$

where $x_{i\omega}=x_i(t-\omega)$, $i=1,2,3$. According to (5) and (21), we can have

$$F(x_\omega,y(t),z(t))=\begin{pmatrix} -\beta_1 & \beta_1 & 0 & 1 \\ \beta_2-y_3(t) & \beta_3 & -(c_1(x_\omega)x_{1\omega}+z_1(t)) & 0 \\ c_2(x_\omega)x_{2\omega}+z_2(t) & y_1(t) & -\beta_4 & 0 \\ 0 & y_3(t) & c_2(x_\omega)x_{2\omega}+z_2(t) & \beta_5 \end{pmatrix}$$

$$G(x_\omega,y(t),z(t))=\begin{pmatrix} d_5 & -\beta_1 & -(c_2(x_\omega)x_{2\omega}+z_2(t)) & 0 \\ L_1 & -\beta_3-d_6 & c_1(x_\omega)x_{1\omega}+z_1(t) & -y_3(t) \\ 0 & -y_1(t) & 0 & 0 \\ -1 & 0 & -(c_2(x_\omega)x_{2\omega}+z_2(t)) & -\beta_5-d_7 \end{pmatrix}$$

where $L_1=y_3(t)-\beta_2$, $0<d_5<\beta_1, d_6>0, d_7>0$, we can obtain

$$F(x_\omega,y(t),z(t))+G(x_\omega,y(t),z(t))=-N(x_\omega,y(t),z(t))$$

Where $N=[n_{i,j}]\in R^{d\times d}$, $i\neq j, n_{i,j}=-n_{j,i}$, $i=j, n_{i,j}\in R^{+}$, $i,j=1,2,3,4$.

Hence, the GFPLS with different dimension (d>c) is realized based on proposition 1.

When $\omega=0.05, d_5=25, d_6=5, d_7=4$,

$$C(x_\omega)=\begin{pmatrix} x_{2\omega} & & \\ & x_{1\omega}-x_{3\omega} & \\ & & x_{1\omega}/2 \\ & & 2.5-x_{2\omega} \end{pmatrix}$$

The error state curves of GFPLS with different dimension (d>c) are shown in Fig. 8.

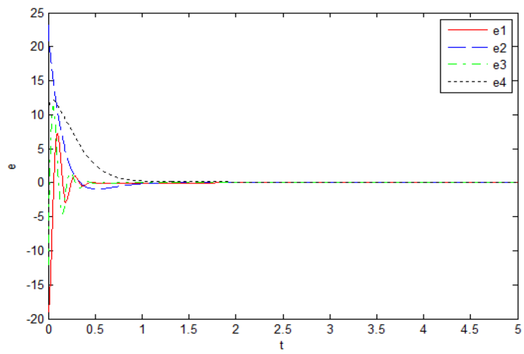

Fig. 8. The error state curves of GFPLS with different dimension (d>c).

IV. Conclusions

In this paper, the GFPLS of fractional-order chaotic system is investigated based on the fractional-order stability theorem. This synchronization scheme is more complex and generalized than the existing synchronization method. The numerical simulations with the same and different dimension are given to illustrate the effectiveness of the proposed method. In the future, GFPLS has more potential to be applied in the secure communication.

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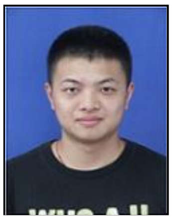

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