Estimation of RC Global Interconnect Slew in 0.18μm Technology Using Inverse Gamma Distribution Function

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***Abstract*—As the technology is shrinking towards the ultra deep sub micrometer regime, timing verification of digital integrated circuits becomes an extremely difficult task due to statistical variations in the gate and wire delays. Statistical timing analysis techniques are being developed to tackle this problem. The variations of critical dimensions in modern VLSI technologies lead to variability in interconnect performance that must be fully accounted for the timing verification. However, handling a multitude of inter-die/intra-die variations and assessing their impacts on circuit performances can dramatically complicate the timing analysis. For optimizations like physical synthesis and static timing analysis, efficient interconnect delay and slew computation is critical. Slew indicates the rate of change of input and output signals. Slew rate determines the ability of a device to handle the varying signals. Determination of slew rate to a good proximity is thus very much essential for efficient design of high speed CMOS integrated circuits as the increase in waveform slew directly enhances the delay of the interconnections. This work presents an accurate and efficient model to compute the slew metric of on-chip interconnect of high speed CMOS circuits. Our slew metric assumption is based on the Inverse Gamma Distribution Function. The inverse gamma distribution is used to characterize the normalized homogeneous portion of the step response. For a generalized RC interconnect model, the stability of the Inverse Gamma Distribution model is guaranteed. The accuracy is proved by comparing our approach with the established methods and SPICE results. It is shown that our approach could result an error in slew calculation as low as 1% with lower value of driver resistance when compared with the SPICE results.**

***Index Terms*—Moment matching, on-chip interconnect, probability distribution function, slew calculation, VLSI.**

I. Introduction

Complex integrated systems on a single chip require communication between several components on the chip.

Wires, buses or complex networks are used to transmit signals between subsystems. Interconnect delay computation is a critical task, which may be executed millions of times during floor planning, placement, routing etc. So an efficient, highly accurate and closed-form delay and slew metrics are very important for IC designs. Modern chip designs contain an overwhelmingly large number of interconnects that must be analyzed efficiently. As such, efficiency of interconnect analysis is critical in a statistical timing flow. The advances in technology that result in scaled, multi-level interconnects may address the wire-ability problem, but in the process create problems with signal integrity and interconnect delay. Elmore [1] proposed the impulse response of a linear circuit as a probability distribution function (PDF), using the mean of the impulse response to approximate the 50% delay of the circuit, which is the median of the impulse response under the probability interpretation under a step excitation. The Elmore delay metric has been incredibly popular because it is simple, closed-form, and easy to evaluate. However with development of the technology, interconnect delay is becoming comparable in value to cell delay or even dominates it. So, in order to analyze the high speed VLSI circuit a priory, much more accurate interconnect delay and slew metrics are desired. AWE [2] can approach towards SPICE-like accuracy by computing and matching higher order moments of the impulse response, but AWE does not provide any closed-form formula, in particular it involves finding a solution of a non-linear equation. So, a new delay metric is desired which should be highly accurate but also simple and closed-form. As technology scales down, transistor density in the chip is increasing, the length of the interconnect is getting longer [3]. So, efficient and accurate computation of slew metric is crucial for enhancing the switching speed of nano devices. In the nanotechnology age, as ultra deep sub-micron effects continue to wreak havoc on the integrity of the signal, so efficient and accurate computation of the slew metric has become critical.

In this paper, we present a closed form slew metric based on the inverse gamma distribution. Matching the circuit characteristics to that of the parameters of gamma distribution function produces an explicit closed form expression for slew calculation. Our approach is different with respect to the proposals made in [1], [4] in that our slew calculation does not require any look-up table. We have proposed the slew metric, IGSM (Inverse Gamma Slew Metric) using the first two moments of the impulse response. The effectiveness and accuracy of the gamma metric is justified on nets from an industrial design. We have compared with [5] which is based on Weibull distribution and found that the proposed approach is more accurate in capturing and estimating the slew metric for RC global interconnect.

II. Basic Theory

A. Moments of the Linear Circuit Response

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Applying a Taylor series expansion of e^{-st} about s = 0 yields [6],

$$\left.\begin{aligned} H(s) &= \int_0^\infty h(t)\left\{1 - st + \frac{1}{2!}s^2t^2 - \frac{1}{3!}s^3t^3 + \ldots\right\}dt \\ &= \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} s^i \int_0^\infty t^i h(t)dt \end{aligned}\right\} \quad (1)$$

The i^{th} circuit-response moment, $\tilde{m}_i$ is defined as [2]:

$$\tilde{m}_i = \frac{(-1)^i}{i!}\int_0^\infty t^i h(t)dt \quad (2)$$

From (2) and (3), the transfer function H(s) can be expressed as:

$$H(s) = \tilde{m}_0 + \tilde{m}_1 s + \tilde{m}_2 s^2 + \tilde{m}_3 s^3 + \ldots \quad (3)$$

B. Central Moments

Similar to moments, central moments are distribution theory concepts. Following Elmore's distribution function analogy, we can use them to explain the properties of Elmore delay approximation. Consider the moment definition given again:

$$\tilde{m}_q = \frac{(-1)^q}{q!}\int_0^\infty t^q h(t)dt \quad (4)$$

The mean of the impulse response is given by [4], [6],

$$\mu = \frac{\int_0^\infty th(t)dt}{\int_0^\infty h(t)dt} = \frac{-m_1}{m_0} \quad (5)$$

It is straightforward to show that the first few central moments can be expressed in terms of circuit moments as follows [7]:

$$\left.\begin{aligned} &\mu_0 = m_0, \mu_1 = 0, \mu_2 = 2m_2 - \frac{m_1^2}{m_0}, \\ &\mu_3 = -6m_3 + 6\frac{m_1 m_2}{m_0} - 2\frac{m_1^3}{m_0^2} \end{aligned}\right\} \quad (6)$$

Unlike the moments of the impulse response, the central moments have geometrical interpretations:

μ_0 is the area under the curve. It is generally unity, or else a simple scaling factor is applied.

μ_2 is the variance of the distribution which measures the spread or the dispersion of the curve from the center. A larger variance reflects a larger spread of the curve.

μ_3 is a measure of the skew-ness of the distribution; for a uni-modal function its sign determines whether the mode (global maximum) is to the left or to the right of the expected value (mean). Its magnitude is a measure of the distance between the mode and the mean.

C. Higher Order Central Moments in RC Trees

The second and third central moments are always positive for RC tree impulse responses [7]. The positive ness of the second order central moment is obvious from its definition;

$$\mu_2 = \int_0^\infty (t-\mu)^2 h(t)dt \quad (7)$$

The impulse response, $h(t)$, at any node in an RC tree is always positive. Hence, the second central moment μ_2 is always positive.

D. Moments of Probability Density Function

A probability function is a real valued set function where the domain is a subset of the sample space, *S*, and the range is a real number in the interval [0],[1]. Generally, a function $P_r\{*\}$ should satisfy the three Kolmogorov axioms [8], or equivalent conditions, in order to be considered as a probability function:

$$\left.\begin{aligned} &(i) P_r\{s\} = 1; \\ &(ii) P_r\{A\} \geq 0 \textit{ for } \text{all } A \in S; \\ &(iii) P_r\{A \cup B\} = P_r\{A\} + P_r\{B\} \\ &\text{if } A \cap B = \varphi, A \in S, B \in S \end{aligned}\right\}$$

The distribution function of a continuous random variable *T* denoted by $F_T(t)$ provides the value of $P_r\{T \leq t\}$ for any real number $-\infty \leq t \leq \infty$. The associated probability density function (PDF), denoted by $f_T(t)$ is the derivative of the distribution function with respect to *t*, thus,

$$\left.\begin{aligned} &f_T(t) = \frac{dF_T(t)}{dt} \\ &\textit{and } F_T(t) = \int_{-\infty}^{t} f_T(\tau)d\tau \end{aligned}\right\} \quad (8)$$

The median, *t (0.5)*, is defined by:

$$P_r\{T \leq t_{(0.5)}\} = F_T(t_{(0.5)}) = \int_{-\infty}^{t_{(0.5)}} f_T(t)dt = 0.5 \quad (9)$$

Whereas, the expected value or mean, $E(t)$ of a continuous random variable *T* with distribution $f_T(t)$ is:

$$E[T] = \int_{-\infty}^{\infty} t f_T(t)dt \quad (10)$$

The mean is also the first moment of the distribution (or PDF). In general, the i^{th} moment m_i of the distribution is:

$$m_i = E[T^i] = \int_{-\infty}^{\infty} t^i f_T(t)dt \quad (11)$$

Note that we use '~' to distinguish between the probability moments m_1, m_2 and circuit moments $\tilde{m}_1, \tilde{m}_2$.

E. Relation between Probability Density Functions and Circuit Response

Any function $f(t)$ can be treated as a probability density function if it is defined in the range [a, b] and satisfies

$$\left.\begin{aligned} &f(t) \geq 0 \forall t \\ &\int_a^b f(t)dt = 1 \end{aligned}\right\} \quad (12)$$

If $f(t)$ is equal to zero outside of the range [a, b], we can replace the integration limits in (13) with $-\infty$ and ∞ . Elmore [1] was the first to apply moments for delay approximation of a limited class of circuit responses by observing that the impulse response of a circuit can be treated as a probability density function. He used this observation to justify the approximation of the 50% point of a monotonic step response (the median point of the impulse response) by the first moment (mean of the impulse response). It was shown that the impulse response corresponding to an RC tree is unimodal with positive skew [7]. From this it follows that the mode is less than the median which is less than the mean and vice versa [8],[9]: i.e.

(Skew > 0) if and only if (mode <median < mean)

This proved that the Elmore delay is an upper bound for the 50% step response delay, and was shown to hold for finite input signal rise time. An important observation is that because of the variation in impulse response shapes along an interconnect path, the relative accuracy of the Elmore delay bound can be quite poor. Especially for the interconnects associated with deep submicron technologies, more than one moment is needed to capture the waveform shape-characteristics.

III. Proposed Delay Model

Elmore's model is based on the analogy between non-negative impulse responses and probability density functions. In theory, Elmore's distribution interpretation can be extended beyond simply estimating the median by the mean if higher order moments can be used to characterize a representative distribution function. Once characterized, the delay can be approximated via closed form expression or table lookup of the median value for the representative distribution family. The proposed slew model is based on the inverse gamma distribution. The inverse gamma distribution is a two parameter continuous distribution [10]. It is well suited to match the impulse response of the generalized RC network since both are unimodel and have non-negative skewness. The inverse gamma distribution's probability density function is defined as,

$$f(x;\alpha,\beta)=\frac{\beta^{\alpha}}{\Gamma(\alpha)}\left(\frac{1}{x}\right)^{\alpha+1}\exp\left(-\frac{\beta}{x}\right) \tag{13}$$

with shape parameter α and scale parameter β.

Mean for the inverse gamma distribution is given by,

$$Mean(E[x])=\frac{\beta}{\alpha-1} \tag{14}$$

Variance is given by,

$$Variance=\frac{\beta^2}{(\alpha-1)^2(\alpha-2)} \tag{15}$$

Mode is given by,

$$Mode=\frac{\beta}{\alpha+1} \tag{16}$$

A. Calculation of Parameters of the Inverse Gamma Distribution Function

Since inverse gamma distribution has two parameters *α* and *β*, so by using moment generating function completely characterizes this model. Hence, the parameters *α* and *β* of inverse gamma distribution can be presented in terms of moments. Moment generating function of inverse gamma distribution is given as below.

$$E[X^n]=\frac{\beta^n}{(\alpha-1)(\alpha-2)\ldots\ldots\ldots\ldots(\alpha-n)} \tag{17}$$

When *n*=1,

$$E[X]=\frac{\beta}{\alpha-1} \tag{18}$$

When *n*=2,

$$E[X^2]=\frac{\beta^2}{(\alpha-1)^2(\alpha-2)} \tag{19}$$

Hence, we can write,

$$m_1=\frac{\beta}{\alpha-1} \tag{20}$$

$$m_2=\frac{\beta^2}{(\alpha-1)^2(\alpha-2)} \tag{21}$$

From (20) and (21),

$$(\alpha^2-3\alpha+2)m_2=(\alpha-1)^2 m_1^2 \tag{22}$$

After solving above equations

$$\alpha^2(m_2-m_1^2)+\alpha(2m_1^2-3m_2)+(m_2-m_1^2)=0 \tag{23}$$

By solving (23), we get,

$$\alpha=\frac{-(2m_1^2-3m_2)\pm\sqrt{(2m_1^2-3m_2)-4(m_2-m_1^2)(2m_2-m_1^2)}}{4(m_2-m_1^2)} \tag{24}$$

By taking Positive sign we get for α<1/2

$$\alpha=\frac{4m_2-2m_1^2}{4(m_2-m_1^2)} \tag{25}$$

From (20) and (25), we can write for *β* as,

$$\beta=\frac{2m_1^3}{4(m_2-m_1^2)} \tag{26}$$

B. Proposed Closed Form Slew Metric

The step response of an RC circuit is a cumulative density function (CDF) [6]. The RC response is considered as a single-pole exponential waveform and can be modeled as, $h(t)=1-e^{-\frac{t}{\beta}}$, $t>0$, If h(t) satisfies the following conditions:

$$0\le h(t)\le 1 \text{ and } \lim_{t\to-\infty} f(t)=0, \lim_{t\to\infty} f(t)=1 \tag{27}$$

Now, let T_{LO} and T_{HI} be 10% and 90% delay points, respectively. Matching to these points to the CDF yields,

$$0.1 = 1 - e^{-\frac{T_{LO}}{\beta}} \quad (28)$$

$$0.9 = 1 - e^{-\frac{T_{HI}}{\beta}} \quad (29)$$

From (28) and (29), we have,

$$T_{LO} = \lambda \ln\left(\frac{10}{9}\right) = 0.1053\beta \quad (30)$$

$$T_{HI} = \lambda \ln(10) = 2.302\beta \quad (31)$$

Using equations (30) and (31), we define the inverse gamma slew metric (we call this metric as IGSM) as,

$$IGSM = T_{HI} - T_{LO} = 2.1976\beta \quad (32)$$

Using (32), we can write the closed form expression of the slew metric in terms of first two circuit moments as,

$$IGSM = \frac{4.3952 m_1^3}{4\left(m_2 - m_1^2\right)} \quad (33)$$

From the above derived equation (33), for the slew metric for the on-chip interconnect using inverse gamma distribution function, we see that the slew metric function is a simple function of the two first circuit moments. This is our proposed closed form slew model.

IV. Simulation Results

In order to verify the efficiency of our model, we have extracted 208 routed nets containing 1026 sinks from an industrial ASIC design in 0.18 μm technology. We choose the nets so that the maximum sink delay is at least 10 ps and the delay ratio between closest and furthest sinks in the net is less than 0.2. It ensures that each net has at least one near end sink. We classify the 1026 sinks into the following three categories:

- 511 far-end sinks have delay greater or equal to 75% of the maximum delay to the furthest sink in the net.
- 343 mid-end sinks which have delay between 25% and 75% of the maximum delay and,
- 172 near-end sinks which have delay less than or equal to 25% of the maximum delay.

For each sink we computed the slew using SPICE simulator. We compared our slew metric with [1], [4], [5] and [11]. We call these metrics as EDS, BakS and WbS, GSM, respectively. The comparison of our slew metric (IGSM) with BakS, WbS, GSM and EDS is shown in Table 1 to Table 4. From the results shown, we find that our proposed model provides the best slew estimation compared to other approaches and results an average error of less than 2% for lower value of driver resistance and an average 5% for higher value of driver resistance.

TABLE I: Average (%) Relative Error with Diver Resistance = 0Ω

sinks	BaKs	EDS	WbS	GSM	IGSM
Near	65.45	786.13	43.72	39.34	38.24
Mid	11.76	24.27	4.65	3.91	4.17
Far	9.23	11.23	2.831	1.987	2.087
Total	9.23	11.23	2.831	1.987	1.943

TABLE II: Average (%) Relative Error with Diver Resistance = 100Ω

sinks	BaKs	EDS	WbS	GSM	IGSM
Near	17.25	143.23	15.34	39.34	28.14
Mid	12.3	31.2	7.87	3.91	3.89
Far	9.45	16.6	6.78	1.987	1.76
Total	10.35	29.4	7.23	1.987	1.89

TABLE III: % Standard Deviation with Driver Resistance =0Ω

sinks	BaKs	EDS	WbS	GSM	IGSM
Near	44.25	615.1	27.38	19.63	18.78
Mid	7.832	23.62	4.59	3.79	3.47
Far	6.96	10.30	3.1	2.78	2.69
Total	6.96	10.30	3.1	2.78	2.75

TABLE IV: % Standard Deviation with Driver Resistance =100Ω

sinks	BaKs	EDS	WbS	GSM	IGSM
Near	18.3	98.9	15.66	19.63	18.23
Mid	10.23	26.56	7.67	3.79	3.73
Far	7.12	78.65	6.98	2.78	2.87
Total	10.54	78.65	6.98	2.78	2.67

V. Conclusion

We have proposed Inverse Gamma Distribution function based closed form Slew Metric model for the RC trees that is a simple function of two moments of impulse response. Our model has Elmore delay as upper bound but with significantly less error. The novelty of our approach is justified by the calculated slew from the experiments performed on the industrial nets. For a generalized RC interconnect model the stability of the homogeneous Inverse Gamma Distribution model is guaranteed.

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