

Design Discrete Frequency Coding Waveform Based OFDM for MIMO-SAR

Wael Mehany, Licheng Jiao, and Xiangrong Zhang

Abstract—Synthetic Aperture Radar (SAR) has been widely used for a well-proven remote sensing technique for more than 30 years. However, single-antenna SAR systems cannot achieve wide-swath and high-resolution imaging at the same time. Multi-Input Multi-Output (MIMO) radar can be used to form a synthetic aperture for high resolution imaging. To successfully utilize the MIMO-SAR system for practical imaging application, the orthogonal waveform design plays a critical role in image formation. In this paper, we modify the transmitted signal of the discrete frequency coding waveform with linear frequency modulation (DFCW-LFM) for MIMO-SAR from fixed chirp rate to variable chirp rate. Changing between the up –and down-chirp waveform can integrated between DFCW and Orthogonal Frequency Division Multiplexing (OFDM) to improve integrated side-lobe level ratio (ISLR) and peak side-lobe level ratio (PSLR) for MIMO-SAR. Modified genetic algorithm (GA) is proposed to numerically optimize the DFCW based OFDM. Some of the designed results are presented.

Index Terms—MIMO-SAR, SAR applications, orthogonal waveform design, discrete frequency-coding waveform, OFDM, genetic algorithm.

I. INTRODUCTION

Multi-input multi-output (MIMO) Synthetic aperture radar (SAR) promises improved performance over conventional SARs [1]. The optimal design of orthogonal waveform with low autocorrelation peak side lobe level ratio (A-PSLR) and cross correlation peak side lobe level ratio (C-PSLR) is crucial for implementing multi-input multi-output (MIMO) radar systems [2]. The orthogonal waveforms used by the MIMO radar systems must be carefully designed to avoid the self-interference and to achieve high range resolution; low aperiodic autocorrelation ensures high range resolution, high signal to noise ratio (SNR) and high resolution of multiple targets [3]. Orthogonal coding signals including orthogonal phase coding signal, orthogonal discrete frequency coding signals have attracted attention in the design of MIMO radar. The discrete frequency-coding waveform (DFCW) has been widely used as wideband radar signal to allow high range resolution and improve detection capability [4].

The autocorrelation peak side-lobe level ratio (A-PSLR) of discrete frequency-coding waveform with fixed frequency pulses (DFCW_FF) is large enough [5], [6]. Replacing the fixed frequency pulses with linear frequency modulated (LFM) pulses may lower the A-PSLR. However, the grating lobes may appear in the range response if $T\Delta f < 1$, where T is

the subpulse duration and Δf is the frequency step. Deng proposed a simulated annealing (SA) algorithm to design DFCW with DFCW_FF and presented some design results [7]. In [8], Levanon proposed an approach to nullify the grating lobes for step- frequency pulses. In [9] Liu employ the method proposed in [8] to mitigate the ASP for DFCW with LFM pulses and nullify the grating lobes.

In ground-based surveillance MIMO radar to ensure weak targets are not covered by strong ones, it is only necessary to consider the peak side-lobe level ratio (PSLR) performance of the signals. But in MIMO-SAR system, as SAR imaging is the process of adding a large number of scatter signals, to ensure the weak scattering background is uncontaminated by any adjacent strong scattering area, not only should be PSLR be lower but the integrated side-lobe level ratio (ISLR) should also be maintained at a lower level [10]. In MIMO-SAR systems, the ISLR is doubly influenced by the energy of the autocorrelated side lobe and the energy of cross-correlation.

Based on the research above, this paper investigates a cost function contains ISLR and PSLR for the design of DFCW with modified LFM based OFDM pulses sets that can be used in MIMO-SAR systems. By changing modified LFM pulses between up and down chirps, we can take the advantages of orthogonal frequency division multiplexing (OFDM) to improve ISLR and PSLR for MIMO-SAR. OFDM has been introduced as a good technique for wideband radar pulses [11]. By setting the relationships between the frequency step Δf , LFM bandwidth B and subpulse duration T , we can reduce the autocorrelation sidelobe peak, as well as nullify the grating lobes.

The remainder of this paper is organized as follows. Section II details the proposed signal model for DFCW based OFDM. In section III, we introduce a modified genetic algorithm (GA) to numerically optimize the DFCW based OFDM. Design results are presented in Section IV, whereas the conclusions are drawn in Section V.

II. DFCW WAVEFORM DESIGN BASED OFDM

The DFCW based OFDM-coded signals set consisting of L signals can be represented as

$$\left\{ S_l(t) = \sum_{n=0}^{N-1} p_n^l(t - nT_p) \cdot e^{j c \pi k t^2} \right\}, \quad l = 1, 2, \dots, L \quad (1)$$

where

$$p_n^l(t) = \begin{cases} e^{j 2 \pi f_n^l t}, & 0 \leq t \leq T_p \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

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Wael Mehany, Licheng Jiao, and Xiangrong Zhang are with School of Electronic Engineering, Xidian University, Xi'an, China (e-mail: waelelfarra@yahoo.com).

And f_n^l is the coding frequency of subpulse n of waveform l in the waveform set. The coding frequency sequence $\{f_1^l, f_2^l, \dots, f_N^l\}$ for waveform l ($1 \leq l \leq L$) of a DFCW based OFDM set is restricted to be a permutation of $\{0, \Delta f, 2\Delta f, \dots, (N-1)\Delta f\}$, and Δf is normally chosen to nullify the grating lobes. For convenience, a coding frequency sequence $\{n_1\Delta f, n_2\Delta f, n_3\Delta f, \dots, n_N\Delta f\}$ of a DFCW based OFDM set is simply represented with the coefficient sequence $\{n_1, n_2, n_3, \dots, n_N\}$ which is a unique permutation of sequence $\{0, 1, 2, \dots, N-1\}$, k is the frequency slope, related to the bandwidth of the signal pulse $B=kT$ and c is the chirp rate sign(+ for up chirp, - for down chirp). The choice of BT , $B/\Delta f$ and $T\Delta f$ are very important for the waveform design, these parameters relationships are mentioned in Table I [9]. The amplitude and frequency of one of the DFCW based OFDM set are shown in Fig.1(a) and Fig. 1(b).

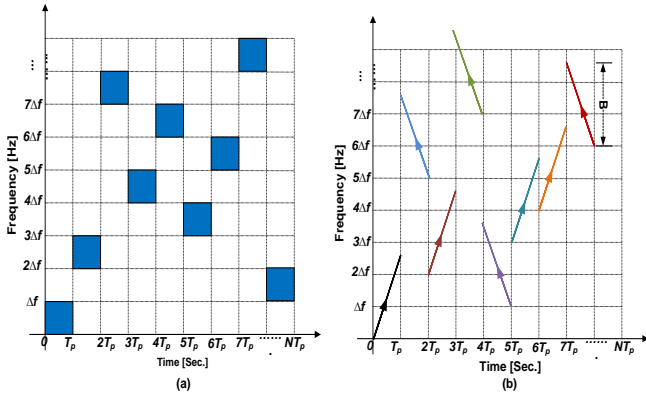


Fig. 1. Example of DFCW based OFDM waveform (a) Amplitude, (b) Frequency.

TABLE I: LENGTH OF FIRING SEQUENCES AND RELATED PARAMETERS

N	$T\Delta f$	BT	$B/\Delta f$
16	2	24	12
16	3	36	12
32	3	72	24
48	3	108	36
16	3.5	40	11.5

By changing LFM to MLFM, we get DFCW_MLFM set

$$\left\{ S_l(t) = \sum_{n=0}^{N-1} p_n^l(t - nT_p) \cdot e^{j\pi k t^3} \right\}, \quad l = 1, 2, \dots, L \quad (3)$$

The optimal design of an orthogonal signal requires the autocorrelation function of the desired signal to have the side lobe as low as possible and the value of the cross-correlation function to be as small as possible, the aperiodic autocorrelation function of the coding signal S_l can be defined as

$$ACF(S_l, \tau) = \frac{1}{N} \int_t S_l(t) S_l^*(t - \tau) dt, \quad l = 1, 2, \dots, L \quad (4)$$

The cross-correlation function of the coding signal S_i and S_j can be defined as

$$CCF(S_i, S_j, \tau) = \frac{1}{N} \int_t S_i(t) S_j^*(t - \tau) dt, \quad (5)$$

$$i \neq j, \quad i, j = 1, 2, \dots, L$$

For the design of orthogonal DFCW based OFDM code sets used in MIMO-SAR, a good approach is to numerically search the best sequences by minimizing a cost function that measures the degree of a specified result meets the design requirements. A traditional optimization method is to minimize the maximum autocorrelation sidelobe peaks (ASPs) and maximum cross-correlation peaks (CPs). However, a more stable optimization method is to minimize a cost function containing the total autocorrelation side-lobe energy and cross-correlation energy. In this paper focusing on SAR application, the peak side-lobe level ratio (PSLR) and integrated side lobe level ratio (ISLR) is included in the cost function. In order to improve the quality of the image contrast the performance of the PSLR/ISLR is considered. PSLR is defined as the value of the ratio of the maximum side lobe and main lobe. The definitions of the autocorrelation and cross-correlation PSLR are as follows [12].

$$PSLR_{Al} = 10 \log_{10} \left\{ \frac{\max_{\tau \in \text{side lobe}} |ACF(S_l, \tau)|^2}{\max_{\tau \in \text{main lobe}} |ACF(S_l, \tau)|^2} \right\}, \quad (6)$$

$$l = 1, 2, \dots, L$$

$$PSLR_{Ci,j} = 10 \log_{10} \left\{ \frac{\max_{\tau} |CCF(S_i, S_j, \tau)|^2}{\max_{\tau \in \text{main lobe}} |ACF(S_i, \tau)|^2} \right\}, \quad (7)$$

$$1 \leq i \leq L-1, i+1 \leq j \leq L$$

The ISLR can be defined as the ratio of the integrated energy of all side lobes which spreads across the whole time domain to the integrated energy of the main lobe in the pulse compression function [13]. The synthetic ISLR, which includes the autocorrelation side lobe energy and all the cross-correlation energy, can be expressed as [14].

$$ISLR_s = 10 \log_{10} \left\{ \frac{\int_{\tau} |ACF(S_l, \tau)|^2 + \sum_{i \neq j \leq L} \int_{\tau} |CCF(S_i, S_j, \tau)|^2 - \int_{\tau=-N_r}^{N_r} |ACF(S_l, \tau)|^2}{\int_{\tau=-N_r}^{N_r} |ACF(S_l, \tau)|^2} \right\} \quad (8)$$

where N_r is the first zero location of the main lobe in the pulse compression function, it generally takes twice the 3dB width from the max of main lobe. Hence, the cost function can be defined as

$$CF = \max(PSLR_{Al}, PSLR_{Cp,q}) + w \cdot ISLR_s \quad (9)$$

where w is the weighting coefficient of cost function and is changed to obtain the optimal waveform according to the performance of the PSLR and ISLR. It is clear that a larger w represents a heavier ISLR weight and meantime, a worse PSLR. Our task is to find a w favorable better ISLR with premise of certain PSLR similar to those in literature. The numerical optimization of (9) is a non-deterministic

polynomial (NP)-hard problem. There is no known algorithm that will solve it in polynomial time. The statistical optimization such as GA or SA is much more effective for solving nonlinear optimization problem. In This work, we employed modified GA to minimize the cost function in the design of orthogonal DFCW based OFDM code sets.

III. GA FOR OPTIMIZATION OF DFCW

Genetic algorithms (GA) are robust and adaptive stochastic search techniques for solving optimization problem. A Genetic Algorithm is a search technique used in computing to find true or approximate solutions to optimization and search problems.

Minimizing cost functions (9) is a difficult optimal problem, so the GA can be applied to minimize the cost functions and search the optimal DFCW for given set size and waveform length.

In the optimization process, the individuals are encoded for fired frequency sequences. However, in this case duplicated and omissions will produce due to traditional crossover and mutation of individuals. So, we require to repairing the representation of individuals before applying the crossover and mutation to avoid the duplicates and omissions. To solve this problem we used Grefenstette code to modify the individuals, which is a good method to solve travelling salesman problem in [15], [16]. For example, the following DFCWs with $N=10$ subpulse:

$$S_1 : (1560342798) \quad (10)$$

$$S_2 : (4210857936) \quad (11)$$

Can be coded as Grefenstette codes:

$$S'_1 : (2551221121) \quad (12)$$

$$S'_2 : (5321523311) \quad (13)$$

Frequency firing code	Canonic code	Grefenstette code
1560342798	0123456789	2
1560342798	023456789	2 5
1560342798	02346789	2 5 5
1560342798	0234789	2 5 5 1
1560342798	234789	2 5 5 1 2
1560342798	24789	2 5 5 1 2 2
1560342798	2789	2 5 5 1 2 2 1
1560342798	789	2 5 5 1 2 2 1 1
1560342798	89	2 5 5 1 2 2 1 1 2
1560342798	8	2 5 5 1 2 2 1 1 2 1

Fig. 2. Grefenstette code process.

The Grefenstette coding methods can be described as follows [16], [17]: we take the canonic sequence $S_0:(0123456789)$ as reference code, then remove the first code of S_1 , i.e., "1", and register the code's position index in S_0 , i.e., "2", as the Grefenstette code, then delete the code "1" from S_0 to obtain the updated $S_0:(023456789)$. Now continue

to take out the second code from S_1 to get the code's position index in new S_0 . This process is repeated, until all codes in S_1 are converted to Grefenstette code.

Fig. 2 illustrated this process. In this figure, the first string is the frequency firing code, the second one is the canonic code and the third one is the obtained Grefenstette code.

By using the inverse process we can get easily the frequency firing code "(1560342798)" from Grefenstette code "(2551221121)".

The basic steps to apply the GA algorithm to the code set are summarized as follows [18]. The flow chart for GA processing in the optimization is shown in Fig. 3.

- 1) Generation: Many individual solutions are randomly generated to form an initial population (chromosomes) S_0 .
- 2) Evaluating the fitness function: Obtaining value of each quantity according to (9).
- 3) Select Operator of GA: This step employs a reproduction operator based on roulette wheel selection according to the fitness function value evaluated by step (2).
- 4) Grefenstette coding: Before the conventional crossover and mutation, the individuals' code needs to be changed to Grefenstette code.
- 5) Crossover: Discrete two point crossover is performed, that provides minimal disruption of individual's genes. it is important part for success of GA.
- 6) Mutation: The mutation is performed with a probability ρ_m on a chromosome of the population. The value of a string position is replaced with one of the other $N-1$ values in Grefenstette code matrix when a mutation occurs with probability ρ_m . So some variations can be introduced into the chromosome.
- 7) Switch the Grefenstette code to the frequency code. After the conventional crossover and mutation, the individuals Grefenstette code need to be converted back to the frequency code.
- 8) Termination: The algorithm terminates when a maximum number of generations has been produced, or a satisfactory fitness level has been reached for the population. After reaching an acceptable fitness in a number of generations, the GA processing can be stopped and the GA finish. From above procedure, we can obtain. If termination condition is not satisfied, go back to step (2).

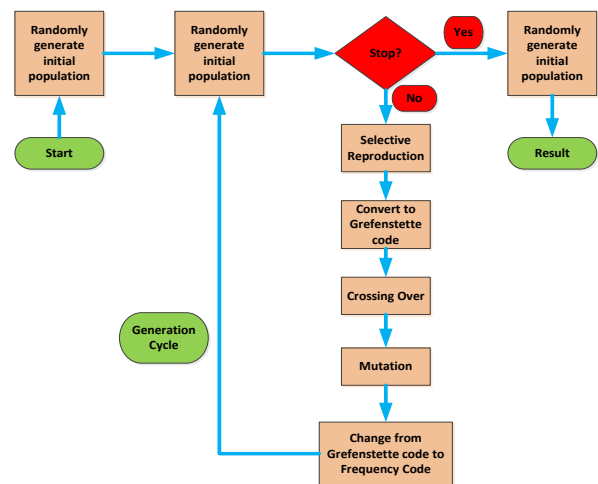


Fig. 3. Block Diagram Representation of Modified Genetic Algorithms (GAs).

V. CONCLUSION

In this paper, we presented a new numerically optimized DFCW sequence code set based OFDM for orthogonal MIMO-SAR system with superior aperiodic correlation in ISLR and PSLR. Good correlation properties in ISLR and PSLR were obtained using a modified genetic algorithm. These results are promising towards improving the performance of the PSLR/ISLR, which directly improves the quality of image contrast in SAR systems. The results show that by changing MLFM pulses between up and down chirps, we can take the advantages of OFDM, and then we improved ISLR and PSLR. The results validate that the proposed approach is indeed an effective solution to improve the SAR image resolution. The effectiveness of the proposed algorithm demonstrated through the design results and compared with the literature values.

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Wael Mehany was graduated from the Military Technical College, Egypt in 1999 with grade "Excellent with Honors". He received the M.Sc. degree in electrical engineering in 2004. He is currently researching the Ph.D. degree in information and communication engineering at School of Electronic Engineering, Xi'an University of Electronic Science and Technology under the guidance of Professor Licheng Jiao. His research interests include MIMO-SAR, Orthogonal Waveform Design, and SAR Applications.

Licheng Jiao received the B.S. degree from Shanghai Jiaotong University, Shanghai, China, in 1982 and the M.S. and Ph.D. degrees from Xi'an Jiaotong University, Xi'an, China, in 1984 and 1990, respectively.

Since 1992, he has been a professor with the School of Electronic Engineering, Xidian University, Xi'an, where he is currently the director of the key laboratory of Intelligent Perception and Image Understanding of the Ministry of Education of China.

He is in charge of about 40 important scientific research projects and has published more than 20 monographs and a hundred papers in international journals and conferences. His research interests include image processing, natural computation, machine learning, and intelligent information processing.

Xiangrong Zhang received the B.S. and M.S. degrees in computer science and technology and the Ph.D. degree in pattern recognition and intelligent system from Xidian University, Xi'an, China, in 1999, 2003, and 2006, respectively. From 2008 to 2010, she was a research fellow with the Centre for Computer Graphics and Visualisation, University of Bedfordshire, Luton, U.K.

She is currently a professor with the Key Laboratory of Intelligent Perception and Image Understanding of the Ministry of Education, School of Electronic Engineering, Xidian University.