Percolation Approach to Digital Comparative Holography via Optical Flow Investigation

M. Hossein Ahmadzadegan and Tapio Fabritius

Abstract—**Digital comparative holography is an important method deployed for working on verifying the body or contortion of two corresponding entities with different micro architecture. The percolation theory is a useful mathematical theory that describes the behavior of connected clusters in a randomly generated graph which can be derived from a picture. Percolation theory is also regarded as a model for displaying a phase transition and it demonstrates the so called critical phenomenon. This in digital comparative holography context implies a drastic change in characteristics. In this paper a novel approach has been taken by utilizing a mathematical theory being the percolation theory to transfer the image achieved by comparative holography in three-dimensional site-percolated form. This operation empowers us to analyze the test object and the behavior of its cluster representation by the help of an image investigation technique being the optical flow investigation. Finally the universality principle will be used to explain and demonstrate the interaction of clusters and how distance together with rotation can remarkably affect the optical flow directions.**

Index Terms—**Digital comparative holography, optical flow, percolation theory, spatial light modulator, universality principle.**

I. INTRODUCTION

Digital comparative holography (DCH) is a method in which comparative holography joins with digital holography [1]. When it comes to comparative holography the conjugated contortions of the master object are reconstructed and the test object is set at light. The observation is carried out in the genuine illumination direction. The distinguished advantage of comparative digital holography compared with conventional comparative holographic interferometry is that the holograms of all states can be kept and later on rebuilt independently from each other. Therefore no more reference waves are desired for a separate coding of the diverse holograms. This property of digital holography leads to the reduction of technical needs for comparative measurements to a large extent. Percolation as a mathematical theory was primarily brought to attention by Broadbent and Hammersley [2], as a probabilistic manner of modeling the flow of a gas or fluid within a porous medium of mini channels that may or may not pass gas or fluid.

This physical concept is mathematically patterned as a three-dimensional network of $n\times n\times n$ vertices, usually called "sites". A site is "filled" with probability p or if edges are removed "blank" with probability 1-p. According to Kolmogorov's zero-one law, for any given p, the probability that an infinite cluster exists is either zero or one. Since this probability is a positively increasing function of p, there must be a critical p symbolized as Pc below which the probability is always 0 and above which the probability is always 1. It is among the patterns demonstrating a phase transition. The occurrence of a critical phenomenon is essential to the appeal of percolation. At this specific point the universality principle states that the value of Pc is associated with the regional structure of the graph, while the behavior of clusters below, at, and above Pc are invariant with respect to the regional structure, and therefore, in some sense are more relevant quantities to take into account. The notion of universality originated primarily from the study of phase transitions in statistical mechanics [3]. A phase transition takes place when the sample witnesses a property alternation in a dramatic way. Phase transitions are recognized by an order parameter such as the visual quality, magnetization or density that changes as a function of a parameter of the system, such as the distance, rotation or temperature. The specific equilibrium value of the parameter at which the system alters its phase is the system's critical point. For systems that demonstrate universality, the closer the parameter is to its critical value, the less sensitively the order parameter depends on the details of the system [4], [5]. In this research work, universality principle is deployed to explain the results of optical flow investigation that is undertaken on the site-percolated test object. Therefore in simple terms, optical flow investigation is going to be used for analyzing the behavior of the clusters in motion after the site-percolation is achieved [6], [7].The concept of optical flow itself was introduced by the American psychologist James J. Gibson in the 1940s to describe the visual stimulus provided to animals moving through the world [8]. Nowadays the term optical flow has been utilized by robotic scientists to highlight that some techniques from image processing and control of navigation, like motion detection, object segmentation, luminance together with the motion disparity are added and incorporated in their findings and productions [8].

II. DIGITAL COMPARATIVE HOLOGRAPHY

In DCH, for measuring the body of an entity the illumination wavelength is changed between taking the two holograms. The transmission of this digital hologram to a test location in Fig. 1 can be done by any data transfer medium. In

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Fig. 1, the hologram is fed into a Liquid Crystal Display as spatial light modulator. A laser rebuilds the hologram optically in Fig 1. The observation is performed from the original illumination direction.

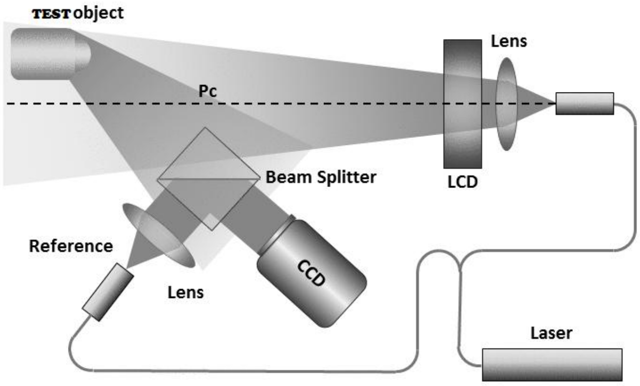


Fig. 1. Digital comparative holography [9], illumination of the test entity with the conjugated wave front of the master.

As it can be observed the chief positive point of comparative digital holography compared with conventional comparative holographic interferometry is that the holograms of all states can be stored safely and at a required moment become reconstructed independently from each other. Therefore no additional reference waves are needed for the separate coding of the holograms [9].

III. Percolation Theory

Percolation theory is the study of how systems of discrete objects are associated with each other. More specifically, percolation is all about the investigation of clusters, their statistics and properties. There are dozens of applications related to percolation theory such as phase transitions (physics), forest fires, epidemics, fracture and many others [10]. Transport characteristics are particularly suited for percolation investigation because transmission or communication among successive neighboring elements is the key. In this research work primarily a Citrus pot is selected as being the sample to work with. It has physical dimensions and one can basically locate it where needed. The color has been fixed to be black and white. This is due to the fact that contrasts are more recognizable and in addition to this it is much easier to study a percolated image in this manner. Since the physical properties and therefore its ways of modellings are interested here, a three-dimensional percolation network of $n \times n \times n$ vertices is ideal for a precise representation. Then having applied the percolation algorithm, the higher contrast sections of the sample which have darker pixels are transferred to the three-dimensional frame. Following this process the entire test object is represented in form of clusters. The test object that is Citrus pot in our case can be observed in Fig. 2.

The three-dimensional representation is demonstrating a site percolation and not a bond one. This is because clusters are not independent. Thus in this case according to theory each and every site is "filled" with likelihood p or if no edges "blank" with likelihood 1-p. Therefore one can see that based on the Kolmogorov's zero-one law, for each and every p, the likelihood that an infinite cluster is out there is zero or one. There must be a Pc then to represent the critical probability, below which the probability is 0 and above which the probability is 1. The Pc describes a phase transition, and the occurrence of a critical phenomenon is essential to the application of percolation. In our case we take the Pc as a point in the middle of the LCD and on the line as it can be seen in Fig. 1. The selected critical point for probability is advantageous because first of all it is of equal distance from the test object and the CCD camera and also it is positioned exactly in the middle of the LCD. Having the site percolated representation together with a strategically located Pc paves the way for having an accurate optical flow investigation and in the same time a smoother justification with universality principle.

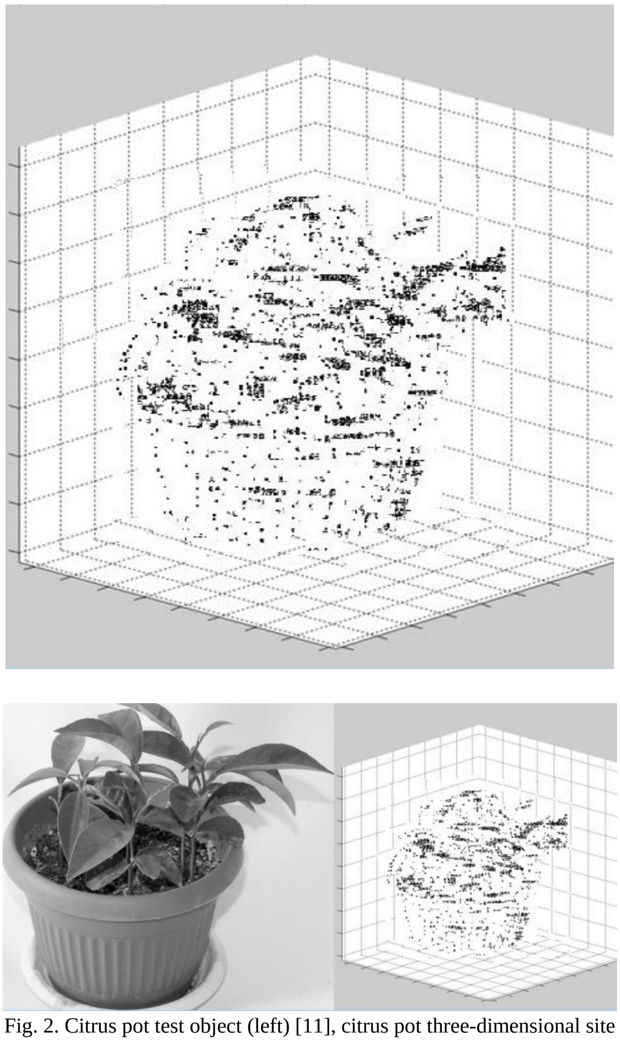


Fig. 2. Citrus pot test object (left) [11], citrus pot three-dimensional site percolation graph (right).

IV. Optical Flow Technique

Optical flow is a model of motion of objects, edges and surfaces in a visual scene that is created through the relative motion between a camera and the scene [12], [13]. The issue of motion investigation or registration among two images is an essential problem that has been widely observed in the literature [14]. One of the chief techniques deployed to address this problem is optical flow, where the pixels of one image are matched and moved to the pixels of the second image that is in our case belonged to the same test object. Therefore, the expected motion vector field hinges on the reference image and thus is not symmetric. Having this in mind, in most applications usually the solutions are not affected by the reference image. One may find symmetrical formulations of the optical flow in [15]-[17], where the solution is bounded to be symmetric deploying mainly a merger of the flow in both directions. As in [18], a symmetric type of formulation of the optical flow has been applied here. Results on the Citrus pot demonstrate the flow with respect to standard optical flow algorithm.

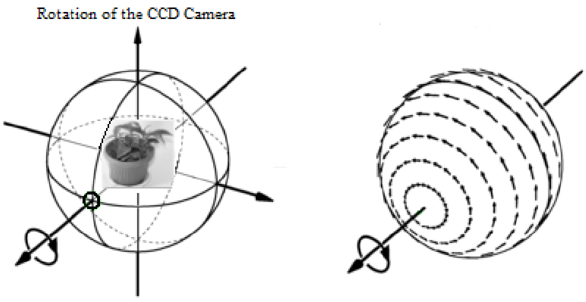


Fig. 3. Three-dimensional optical flow representation of the citrus pot after the motion, rotation and distancing the CCD camera [19].

When the CCD camera is taken away from or moved closer to the Pc, definitely a displacement takes place. Now in order to find the occurred displacement between the two images I_1 and I_2 of the same test object in a symmetric way, the taken measure is to consider an intermediate image I_m exactly situated in the middle between I_1 and I_2, so that there exists a displacement field **f** which fulfills:

$$\forall x,\ I_m(x)=I_1(x-f)=I_2(x+f) \tag{1}$$

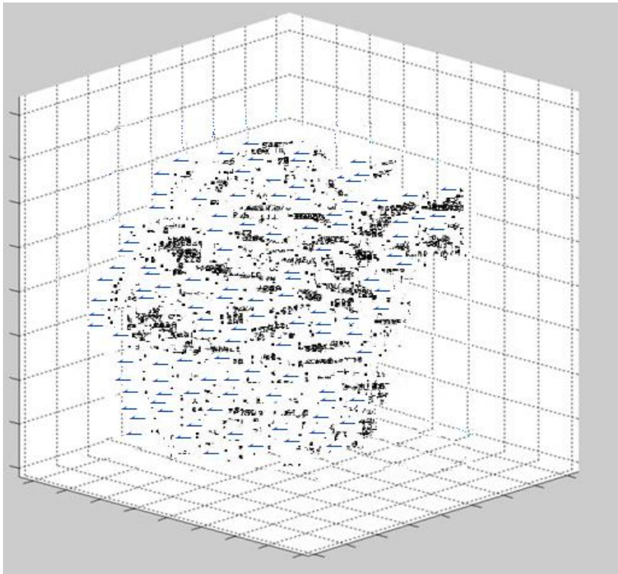
To estimate this displacement, we minimize the energy:

$$E(f)=\int_\Omega(I_1(x^-)-I_2(x^+))^2{}_{dx}+\alpha\int_\Omega\|\nabla f(x)\|^2{}_{dx} \tag{2}$$

where the parameters are denoted $X^+=X+f$ and $X-=X-f$, α is the smoothing coefficient and Ω represents the entire field in which the amount of energy is being minimized and measured. As it can be seen the calculation results in Euler-Lagrange equations which leads to a system of equations that is solved using an iterative Gauss-Seidel scheme. This is beyond the scope of this research work to calculate it numerically but one can observe the technique which is employed here to calculate the displacement. Therefore by minimizing the energy the contrast can be recognizable in an easier manner and thus it further facilitates the calculation of the displacements occurred between the two pictures taken from the same test object after rotation and distancing of the CCD camera.

V. Percolation-Based Analysis via Optical Flow Investigation

In the holographic system, one can control many parameters including the angle, the distance, SLM illumination and also the aperture through which the light travels via CCD camera. In order to begin with the process of investigation, primarily we denote two moments of time being t_1 and t_2. The reason for choosing two moments is twofold. First of all two moments reflect different times and therefore they do not hinge on each other. Secondly at each specific moment a certain action takes place and this aids the investigation to take place seamlessly. Moreover the results can be distinguished with less amount of efforts. The moment t_1 is devoted to Fig. 2 and respectively the moment t_2 is dedicated to Fig. 4. In the moment t_1, the structure and positioning of the holographic system parameters and the test object are in their standard location that is well represented in Fig. 1. From this specific positioning one will get a natural and high quality image like in Fig. 2 demonstrated above. In this moment the test object seems to have some visible three-dimensional forms due to the nature of the holographic imaging. In addition to this one can observe the site-percolated image representing the normal positioning of the clusters.



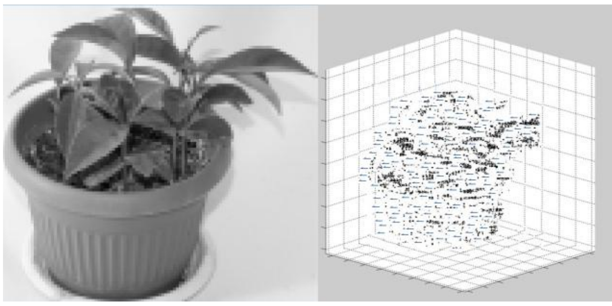


Fig. 4. Drastic drop in citrus pot quality after rotation and distancing the CCD camera.

Now the second stage starts out as the moment t_2 begins. In this moment also the optical investigation is performed which is one of the known digital image analysis techniques. First of all, having the result from the Fig. 2 in mind, in this moment we try to rotate the CCD camera upward as represented in Fig. 3 and keeping the same angle, distance the camera from the selected Pc as shown in Fig. 1.

At this moment the optical flow direction alters as demonstrated in Fig. 3. Now the displacement occurs. One can deploy the proposed formulas (1), (2) to measure the amount of displacements. At this stage the site-percolated representation of the test object should be rechecked. One can observe that the site-percolated clusters after the motion start to move rightward represented with small blue arrows in Fig. 4 and pixels are therefore displaced. Further on in some cases when the neighboring pixels have a drastic color diversity tone difference, they are not well-matched and therefore the picture appears deteriorated and degraded. We have changed the distance of CCD camera from the Pc keeping the same angle and the observation can be counted as very interesting.

According to the universality principle which is one of the percolation theory related notions, this system follows the universality concept in such a way that the closer the parameter is to its critical value, the less sensitively the order parameter depends on the details of the system. This in simple terms implies that as we move the CCD camera closer to the Pc, the resulted image of the test object appears unharmed and has a high quality. Therefore the order parameter has a less sensitivity to the system details. On the other hand, when the CCD camera is taken away and is distancing from the Pc, the picture begins to become degraded and also the pace of deterioration is amplified with a little move toward the laser. Thus this outcome vividly proves that a phase transition has taken place and as a result the critical phenomenon has occurred.

VI. Conclusion

Digital comparative holography is an important method that provides the possibility of a contortion checkout of two corresponding entities. Thus by the help of its deployment the main parameters can be changed and therefore in this manner it facilitates the situation for a comparative analysis. Utilization of this method has made it possible to compare various entities in far situated locations by interferometric sensitivity. In this research paper, we have taken a novel approach by using a mathematical theory regarded as percolation theory to transfer the image obtained by comparative holography in three-dimensional site-percolated form. This operation enables us to investigate the test object and the behavior of its cluster representation by the aid of an image analysis technique being optical flow investigation. Eventually by analyzing the obtained results, we have proved and concluded that the universality principle which is one of the percolation-related notions, can be successfully employed to describe the behavior of clusters resulting to the deterioration and degradation of the initial test object's image.

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