Semianalytic BER Estimation of SC-QPSK under Nakagami-m Frequency Selective Fading Channel with Diversity Reception

Abhijit Bhattacharyya and Neetu Sood

Abstract—**Semianalytic estimation of bit-error-rate (BER) is one of the efficient procedures for evaluating BER for modulation formats having circular constellation with noise component of the decision variable is circularly symmetric with Gaussian distribution. To ensure very rapid simulation run time analytical and simulation techniques are used together unlike conventional Monte Carlo simulation. This technique is demonstrated to estimate BER of SC-QPSK system under Nakagami-m frequency selective fading channel. Finally error rate estimation is extended to system having multiple channel reception using spatial diversity combiners. Our result shows that system performance degrades with increasing value of system parameter.**

Index Terms—**Bit error rate, single carrier–QPSK, Nakgami-m fading channel, diversity.**

I. Introduction

BER estimation has been extensively used as a measurement of system performance nowadays. A trend of computer aided methods for finding BER is widely used because of non availability of an accurate analytical expression for BER. Monte Carlo technique is known as the most general method for estimation of BER. To simulate by this method, a known data sequence is passed through the system and receiver decision device counts the number of bit errors by comparing transmitted and received bits. If a system is processing N bits and it is found that n bits are in error then an unbiased estimator of BER is nothing but sample mean [1]. As N→∞ the estimate of BER will converge to exact BER. The reliability of the estimator is quantified in terms of confidence intervals for finite values of N. Simulation of a system by Monte Carlo technique, operating over a very low BER requires very long simulation run time.

There are several techniques available for accelerating the estimation of BER. The technique used in this paper is semianalytic or quasi-analytic technique. By this technique, the problem is broken into two parts, first to deal with the signal component and next dealing with the noise component of the decision variable. In applying semianalytic technique the combined effect due to the transmitted signal and channel induced inter symbol interference are determined by Monte Carlo simulation and the effect of noise is treated analytically. In [2] Mark T. Core has presented the performance of PSK (Phase Shift Keying) over (DVB-S2) Digital Video Broadcasting - Satellite -Second Generation Channel by semianalytic technique.

Both Rayleigh and Rician fading channels are widely used in wireless communication system studies and research. In [3] Glavieux presented the performance analysis of OFDM-BPSK under Rayleigh and Rician frequency selective fading environment. J. Lu in [4] studied the performance analysis of OFDM-MDPSK in Rician fading channel with diversity reception.

Nakagami-m fading distribution has been drawn the attention of modern researchers because of its ease of manipulation and wide range of applicability to characterise fading amplitude. The performance analysis of a direct sequence code-division multiple-access (DS-CDMA) under Nakagami-m fading channel is presented by Eng and Milstein in [5]. In [6], Alouini and Goldsmith studied the error rate performance of linearly modulated signals over Nakagami-m fading channel by applying moment generating function (MGF) technique. Previous work has assumed that the frequency response of channel is also Nakagami-m distributed with the same fading parameters as the time domain channel [7]. In [8], Zheng Du has presented an accurate analysis which is contrary to the fact that the distribution of samples at the frequency domain can be approximated by Nakagami-m distribution with a new fading parameter different from the time domain fading parameter, which is claimed by Kang et al in [9]. In paper [8], they have shown analytically that error-rate performance of OFDM system under Nakagami-m multipath fading channel does not necessarily improve with increasing fading parameters. Our semianalytic error-rate performance of SC-QPSK system under Nakagami-m frequency selective fading channel also supports this result. In this paper, for further improvement of system performance, spatial diversity combiners are employed at the receiver.

II. Semianalytic BER Estimation of QPSK

Since QPSK constellation consists of four signal points rather than two, semianalytic BER estimator has two dimensions. The signal constellation is illustrated in Figure 1. The transmitted signal points are denoted by S_i, i=1, 2, 3, 4 and the decision regions are denoted by D_i, where i=1, 2, 3, 4. Correct decision is made by the receiver if transmitted signal is S_i and the received signal falls in decision region D_i, otherwise an error occurs. If S_1 is transmitted and the received signal without noise is $\tilde{S}_1$, then an error occurs due to intersymbol interference and distortion, when $S_1 \neq \tilde{S}_1$. It

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The authors are with Dept. of Electronics and Communication Engineering, National Institute of Technology, Jalandhar, India (e-mail: abhijit9800@gmail.com, soodn@nitj.ac.in).

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is $\tilde{S}_1$, which is determined by the semianalytic simulation because simulation is performed only to find the effects of intersymbol interference.

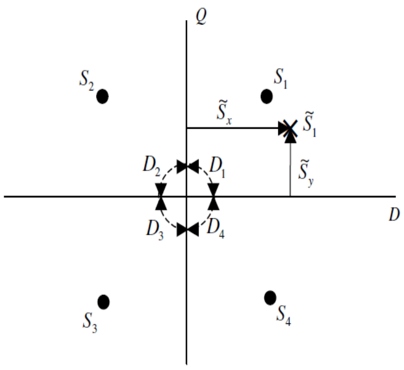

Fig. 1. Semianalytic BER estimation for QPSK.

The direct and quadrature components of $\tilde{S}_1$are defined as $\tilde{S}_x$ and $\tilde{S}_y$ respectively where $\tilde{S}_x = Re\{\tilde{S}_1\}$ and $\tilde{S}_y = Im\{\tilde{S}_1\}$. When noise is taken into account by adding n_x and n_y with $\tilde{S}_x$ and $\tilde{S}_y$ respectively, an correct decision is made by the receiver if $(\tilde{S}_x + n_x, \tilde{S}_y + n_y) \in D_1$. An error occurs when the sum falls outside the decision region D_1. Since we are estimating BER by semianalytic analysis, the effect of noise is determined analytically and does not appear in Figure 1. So if S_1 is transmitted and $\tilde{S}_1$ is received, an error is occurred if

$$\Pr\{Error|S_1\} = \iint_{(\tilde{S}_x+n_x,\ \tilde{S}_y+n_y) \notin D_1} \frac{1}{2\pi\sigma_n\sigma_n} . \exp\left(-\frac{(n_x-\tilde{S}_x)^2}{2\sigma_n{}^2} - \right.$$
$$ny{-}Sy22\sigma n2dnxdny \qquad (1)$$

The conditional error probability bound is given by [10]

$$\Pr\{Error|S_1\} < Q\left(\frac{Re\{\tilde{S}_1\}}{\sigma_n}\right) + Q\left(\frac{Im\{\tilde{S}_1\}}{\sigma_n}\right) \qquad (2)$$

where $Q(.)$ is the Gaussian Q function. By symmetry, the conditional error probability for all four possible transmitted symbols is identical. Suppose S_k is the kth transmitted symbol out of N possible symbols. For each value of k, $1 \le k \le N$, S_k will take one of the four values, S_1,S_2, S_3 and S_4. The conditional symbol error rate bound is

$$\Pr\{Error|S_k\} < Q\left(\frac{Re\{\tilde{S}_k\}}{\sigma_n}\right) + Q\left(\frac{Im\{\tilde{S}_k\}}{\sigma_n}\right) \qquad (3)$$

The overall symbol error rate is nothing but the average of conditional symbol error probability over the entire sequence of N symbols, given by

$$P_S < \frac{1}{N}\sum_{k=1}^{N}\left[Q\left(\frac{Re\{\tilde{S}_k\}}{\sigma_n}\right) + Q\left(\frac{Im\{\tilde{S}_k\}}{\sigma_n}\right)\right] \qquad (4)$$

As in QPSK symbol consists of two bits, bit error rate, P_E, is ${}^{P_S}/_2$.

III. Channel Model

In this paper, to model frequency selective fading channel, we have assumed that channel tap coefficients, $h(n), n = 0, 1, 2, .. L-1$, are mutually independent complex random variable, which can be expressed as

$$h(n) = |h(n)|e^{j\phi(n)} \qquad (5)$$

The channel amplitude, $|h(n)|$ is taken as Nakagami-m distributed random variable with probability density function is given by

$$P_R(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m} \exp\left(\frac{-mr^2}{\Omega}\right), r \ge 0 \qquad (6)$$

where $\Gamma(.)$ denotes Gamma function, $\Omega = \overline{r^2}$ is the average power, m is fading parameter and r is Nakagami distribution envelope. The fading phases, $\phi(n)$ are mutually independent and uniformly distributed over $[0, 2\pi)$, and are independent of fading amplitudes $|h(n)|$'s. Since Nakagami distribution includes comprehensively, scattered, reflected and original version of the transmitted signal. It can be generated using non line-of-sight, $r_{nlos}(t)$ and line-of-sight signal, $r_{los}(t)$ envelopes.

$$r(t) = |r_{nlos}(t)|\exp(1-m) + |r_{los}(t)|(1 - exp(1-m)) \qquad (7)$$

This value of $r(t)$ is used as the envelope of the Nakagami-m faded channel.

IV. Results and Discussion

The BER vs. SNR for QPSK system for two tap frequency selective fading channel has been shown in Figure 2 for different values of *m*. The BER is evaluated using semianalytic estimation. The delay is expressed in terms of the sampling period. Since the simulation sampling frequency is 16 samples per symbol, τ= 8 corresponds to a delay of one-half the sample period. The carrier frequency is considered is 890 MHz and for the simulation of the channel the vehicle speed is considered as 20 meter/sec.

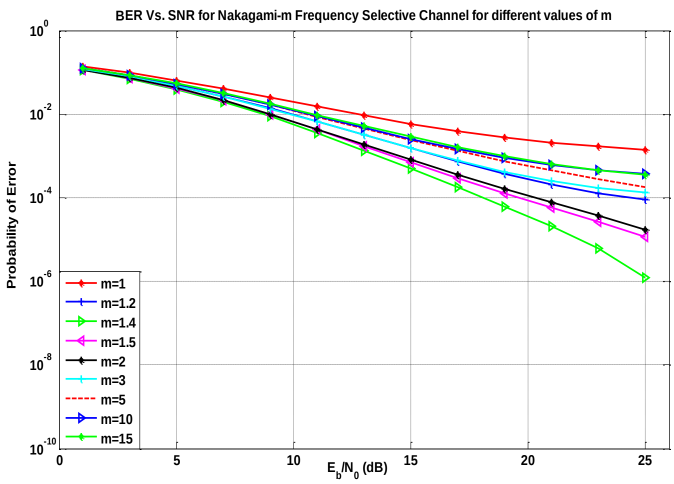

Fig. 2 BER Vs SNR for SC-QPSK system for different values of *m*.

In Fig. 3, the graph of BER versus *m* is plotted. It is reported that optimal value of *m* is 1.4 and probability of error increases for *m* > 1.4. We have taken BER at a fixed

SNR =15dB for different values of *m*. There is a monotonic increase of BER after *m*=1.4. This interesting fact about Nakagami-m channel is already reported by Zheng Du [8].

V. Error Rate Performance Using Diversity Reception

In this section, to mitigate the effect of channel induced ISI spatial diversity combiners are employed. Figure 4 shows the improvement of system performance by using selection, Equal Gain Combiner (EGC) and Maximal Ratio Combiner (MRC) respectively. Optimum value of *m=1.4* is considered for simulating the channel. The delays are taken for two frequency selective channels are delay of 8 and 10 samples respectively. It is evident from Figure 4 that EGC is superior to selection diversity and marginally inferior to MRC diversity.

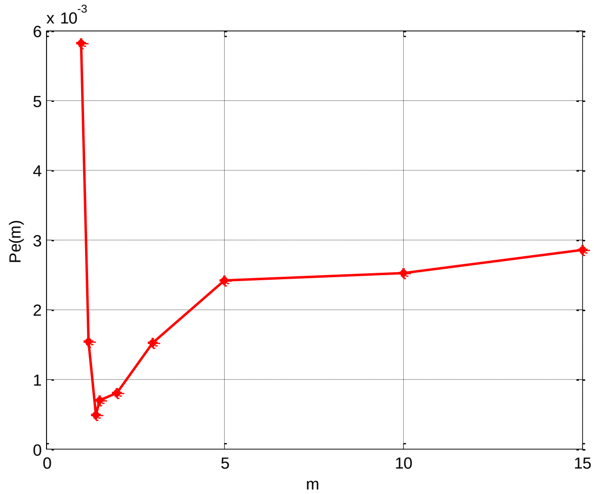

Fig. 3. BER vs. *m* for Nakagami-m Frequency selective channel.

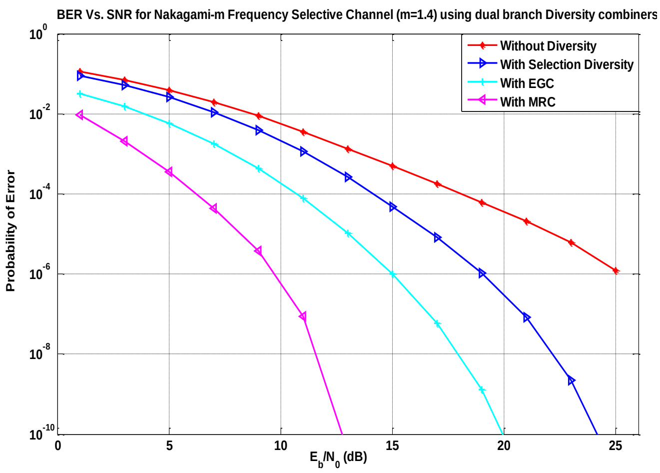

Fig. 4. Comparison of performance of SC-QPSK system using different diversity techniques.

VI. Conclusion

In this paper the performance analysis of single carrier QPSK system in Frequency selective fading channel is presented by using semianalytic method. Here Nakagami-m distributed random variable is decomposed into orthogonal random variables having Gaussian distributed envelopes. The value of *m* for which BER poses least value is 1.4. It is evident from the simulation result that MRC diversity combiner has the best performance among all these three combiners.

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Abhijit Bhattacharyya received B.E with the highest distinction from University of Burdwan, India, in 2006. He is currently pursuing M. Tech from National Institute of Technology Jalandhar, India. His area of interest includes wireless communication and signal.

Neetu Sood received the B.Tech. degree (with Honour) in Electronics and Communication from Sant Longowal Institute of Engineering and Technology, India 2000. In year Jan. 2002, she completed her M.Eng. Degree (With Honour) in Electronics and Communication from Thapar University, Patiala, India and PhD in Electronics and Communication in year 2013. From year 2002 to 2007, she worked with DAV institute of Engineering and Technology, Jalandhar and Guru Nanak Dev University (Regional Campus), Jalandhar.Currently, she is working as Assistant Professor at Dr. B Ambedkar National Institute of Technology, Jalanadhar Her current research includes the simulation of wireless systems based on OFDM and simulation of fading channels.