Speed Control of DC Motor Using Extended Kalman Filter Based Fuzzy PID

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***Abstract*—In this paper, extended Kalman filter (EKF) is used for online optimization of input and output membership functions (MFs) of Mamdani fuzzy PID controller. The proposed controller is employed for controlling the separately excited DC motor. The simulation results show that the fuzzy PID controller with online optimization has better efficiency than classic PID controller and fuzzy PID controller with fixed membership functions.**

***Index Terms*—DC motor, Extended kalman filter, fuzzy PID.**

I. Introduction

Due to the excellent speed control characteristics of DC motors, they are widely used in industry for various applications i.e. robotics, rolling mills, machine tools, position control, mining, paper and textile manufacturing and, etc.[1]. PID control is used as a standard technique for the purpose of controlling DC motors [2]. However, it cannot efficiently cope with the effects of great load variations and changes in model parameters. To overcome these limitations, during the past two decades, various new control techniques, such as Neural Networks, Genetic Algorithm, and Fuzzy Logic are combined with the classic PID to regulate the dc motor speed [3].

The most important advantages of the fuzzy PID (FPID) controllers in comparison with classic PID controllers are: not needing to precise system model, having stable operation even if there is a change in the control parameters and the motor, that result in raising its applications. As it is known, the task of selecting adequate shape and form of fuzzy membership functions (MFs) are complicated procedures and depend on experts' experiences, since there is not a straightforward systematic approach. The typical fuzzy PID controller with fixed MFs cannot adapt itself to a wide range of load changes and the large perturbations imposed by the working environments [4].

In recent years, various methods are presented to conquer these problems aimed at auto tuning the fuzzy MFs. These methods can be divided into two categories: derivative-based methods and derivative free methods [5]. One of the novel and efficient derivative-based methods is the Kalman filter method, which was first presented by Simon [6] for optimizing fuzzy membership function. In this paper, a Mamdani fuzzy PID controller with primary membership function is introduced for controlling dc motor at first. Then, the extended Kalman filter is implemented to online tuning the fuzzy membership function. In fact, EKF estimates the ideal and proper state (MF parameters) for Fuzzy controller based on the current state, current speed error and previous information. Consequently, fuzzy membership functions are updated in each step in order that the motor performance for reference speed following improves.

This paper is organized as follows: In the next section, the system model of a DC motor is formulated. The structure of the Fuzzy PID controller is discussed in Section 3. In Section 4 the EKF method and how it can be used in the fuzzy MF tuning process is explained briefly. Then, in Section 5, the simulation results of the corresponding system are compared with systems controlled with simple PID and non-optimized fuzzy PID controllers.

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II. Mathematical Model of DC Motor

A separately excited DC motor is modeled by the following equations:

$$\frac{di_a}{dt} = \frac{1}{l_a}[v_a(t) - R_a i_a(t) - k_b \omega(t)] \tag{1}$$

$$\frac{d\,\omega(t)}{dt} = \frac{1}{j}\left[k_t i_a(t) - B\,\omega(t) - T_l(t)\right] \tag{2}$$

where $\omega(t)$ is angular speed, $i_a(t)$ is armature circuit current, $v_a(t)$ is motor terminal voltage, $T_l(t)$ is load torque, R_a is armature circuit resistance, B is friction coefficient, k_t is torque coefficient, k_b is voltage coefficient, j is moment of inertia, and l_a is armature circuit inductance.

TABLE I: Motor Parameters

1hp, 220 volts, 4.8 amperes, 1500 rpm		
$R_a = 2.25\Omega$	$l_a = 46.5mH$	$j = 0.07kg.m^2$
$B = 0.002N.m.\frac{sec}{rad}$	$k_t = 1.1N.m/A$	$k_b = 1.1N.m/A$

III. Fuzzy PID Controller

As it is shown in Fig. 1, the fuzzy PID controller is made up of a classic PID controller which its coefficients, such as k_p, k_i, k_d can be calculated using the following equation [7]:

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$$k_a = k_{a\min} + k'_a(k_{a\max} - k_{a\min}) \qquad a \text{ is } p,\ i \text{ or } d \tag{3}$$

where $k'_a \in [0,1]$, is the parameter obtained from the output of the tuning fuzzy controller. $k_{a\max}$ and $k_{a\min}$ are the minimum and maximum values of k_a determined from experiments, respectively.

Fuzzy system is made up of three subcontrollers that each of them tunes the k'_p, k'_i, k'_d parameters separately and independently. Input variables of fuzzy subcontrollers are the error between reference speed and output speed of motor and the derivation of error, and the output variable is k'_a. The inputs domain is in interval $[-1,1]$. This action is done by input normalization. It is presumed that each input has 5 triangular MFs and output has 5 singleton MFs (Figs. 2, 3). Applied inference is Mamdani minimum mechanism and weighted average method is finally used for defuzzification. Hence, the output of each fuzzy subcontroller is calculated as follows:

$$k'_a = \frac{\sum_{l=1}^{M}\left(\min_{i=1}^{2} \mu_{A_i}^{l}(x_i)\right) w_k^l}{\sum_{l=1}^{M}\left(\min_{i=1}^{2} \mu_{A_i}^{l}(x_i)\right)} \tag{4}$$

where $\mu_{A_i}^{l}(x_i)$ is the degree of membership of the ith input, i is the number of inputs, M is the number of fuzzy rules, w_k^l is the centroid of the kth output fuzzy MF for consequent of lth rule. Since each input has 5 MFs, the number of total rules is 25 for each fuzzy subcontroller. If the output speed of motor is ω and the reference speed is ω_r, the error function can be defined as [7]:

$$E = \frac{1}{2}(\omega - \omega_r)^2 \tag{5}$$

The object of employing the Kalman filter is to minimize the error function by online variation of fuzzy membership function parameters. Consequently, the dynamic response of motor improves.

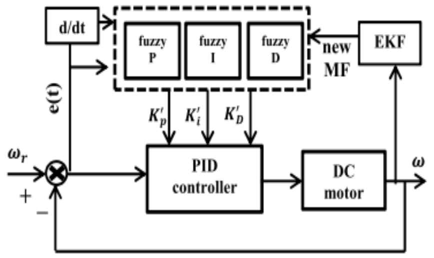

Fig. 1. Structure of the proposed control mechanism

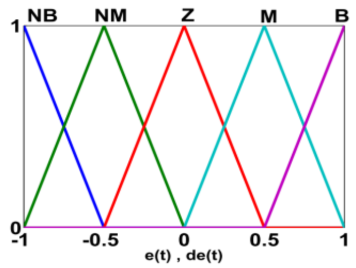

Fig. 2. The initial MFs for the inputs of fuzzy P, I and D controllers

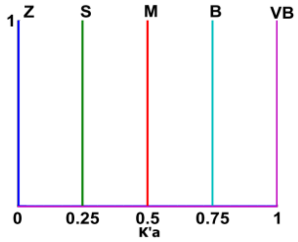

Fig. 3. The initial MFs for the outputs of the three fuzzy P, I and D controllers.

IV. Theory of MF Optimization Via EKF

Consider a nonlinear system represented as follows [5]:

$$x_{n+1} = f(x_n) + w_n \tag{6}$$

$$d_n = h(x_n) + v_n \tag{7}$$

where x_n represents the state of the system at time n. q_n is the process noise, d_n is the observation vector, v_n is the observation noise, and $f(0)$ and $h(0)$ are nonlinear vector functions of the state. The following three equations are recursive estimation equations of the extended Kalman filter:

$$K_n = P_n H_n^T (R_n + H_n P_n H_n^T)^{-1} \tag{8}$$

$$\hat{x}_n = f(\hat{x}_{n-1}) + K_n(d_n - h(\hat{x}_{n-1})) \tag{9}$$

$$P_{n+1} = F_n(P_n - K_n H_n P_n)F_n^T + Q_n \tag{10}$$

where

$$F_n = \left.\frac{\partial f(x)}{\partial x}\right|_{x=\hat{x}_n} \tag{11}$$

$$H_n = \left.\frac{\partial h(x)}{\partial x}\right|_{x=\hat{x}_n} \tag{12}$$

K_n is the Kalman gain, Q_n and R_n are the covariance matrices regarding the process noise q_n and the measurement noise v_n, respectively, P_n the covariance of the prediction error, and $\hat{x}_n$ is the estimated state of the system at time n.

For the EKF training purpose, we let the parameters of MFs constitute the state of the system as:

$$\begin{aligned} x_{(P|I|D)_k} = [&b_{11}^- c_{11} b_{11}^+ .. b_{51}^- c_{51} b_{51}^+ b_{12}^- c_{12} b_{12}^+ ... \\ &b_{52}^- c_{52} b_{52}^+ w_1 .. w_5]_k \end{aligned} \tag{13}$$

where c_{ji}, b_{ji}^- and b_{ji}^+ are the centroid, left half-width and right half-width of the jth triangle MF associated with the ith input, respectively. w_k is the centroid of the kth output fuzzy MF.

The output speed of the motor ω is a nonlinear mapping of the MF parameters. Hence, we can define the nonlinear model of our system as:

$$x_{n+1} = x_n + w_n \tag{14}$$

$$d_n = h(x_n) + v_n \tag{15}$$

where $h(x_n)$ is a nonlinear mapping function between the MF parameters and the motor speed ω. Thus, the EKF recursion in (8) can be applied as:

$$F_n = I \tag{16}$$

$$H_n = \frac{\partial h(x_n)}{\partial x} \equiv \left.\frac{\partial \omega}{\partial x}\right|_{x=\hat{x}_n} = \left[\frac{\partial \omega}{\partial b_{11}^-}\ \frac{\partial \omega}{\partial c_{11}}\ \frac{\partial \omega}{\partial b_{11}^+} \cdots \frac{\partial \omega}{\partial w_5}\right] \tag{17}$$

$$K_n = P_n H_n^T (R_n + H_n P_n H_n^T)^{-1} \tag{18}$$

$$\hat{x}_n = \hat{x}_{n-1} + K_n(d_{n-1} - H_n \hat{x}_{n-1}) \tag{19}$$

$$P_{n+1} = P_n - K_n H_n P_n + Q \tag{20}$$

where I is the identity matrix and H_n is the partial derivative of the motor speed with respect to MF parameters.

V. Simulation Results

The experimental tests are carried out for systems employing a classic PID controller, the fuzzy PID controller with fixed MF, and the proposed online tuning fuzzy PID controller based on EKF. Primarily, we obtain the step response of a dc motor for three controllers. For the online EKF training purpose, It is assumed that Q and R do not change at each iteration, and their value are very small [8] i.e., $q_0 = r_0 = 1e^{-5}$ .Then, the initial covariance of the prediction error is tuned manually by trial and error in order to obtain the mentioned convergence and desired results. For the reference speed of 10 rad/s, as it is shown in Fig. 4, the proposed online tuning fuzzy PID controller gives fast rise time without having overshoot, and the settling time is small when it compared to the responses of two other controllers. In the next stage, three controller responses are obtained regarding to a multi-step speed reference when a full load torque is suddenly added at time 3s. It indicates that, in comparison with, the two other controllers, the speed drop of the system using the proposed controller is obviously smaller. Fig. 5 shows the better efficiency for the system when the proposed controller is used than when PID and fuzzy PID Controllers are exploited. The fuzzy membership functions (MFs) are shown in Figs. 6, 7, 8 after optimization for each of subcontroller k_p, k_i and k_d .

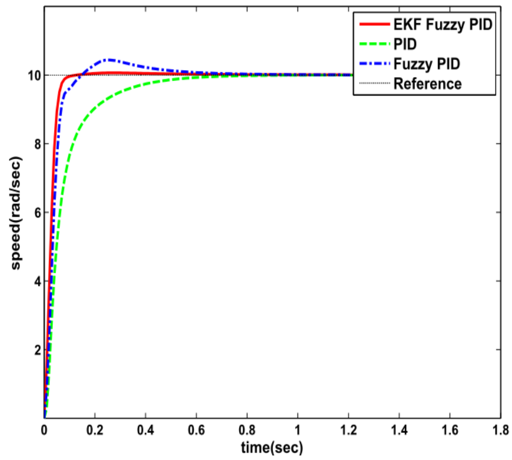

Fig. 4. Step response under no-load starting condition

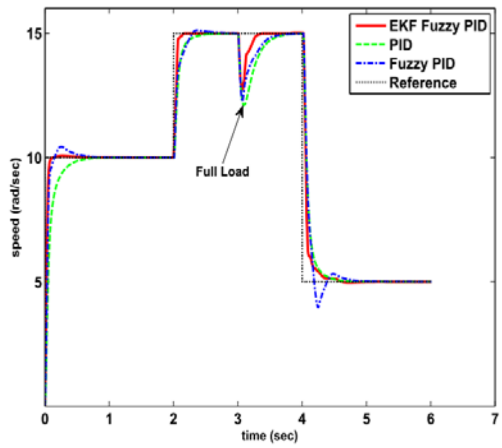

Fig. 5. Speed tracking for multi step speed reference when add load torque suddenly.

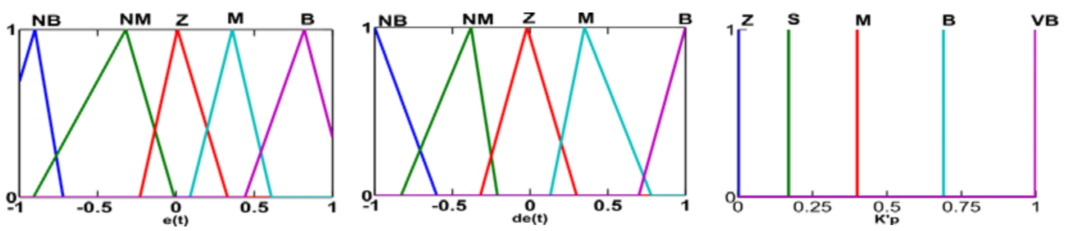

Fig. 6. MFs of the fuzzy P inputs and output

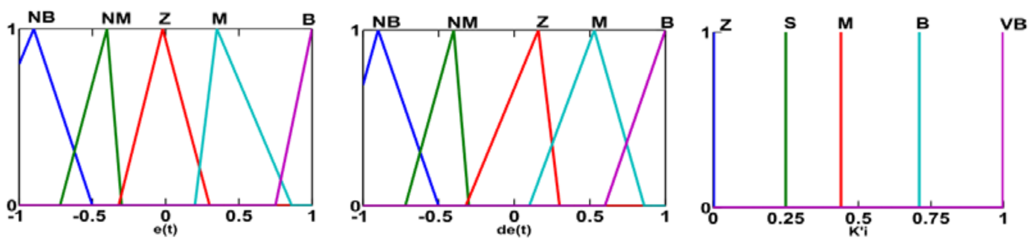

Fig. 7. MFs of the fuzzy I inputs and output after

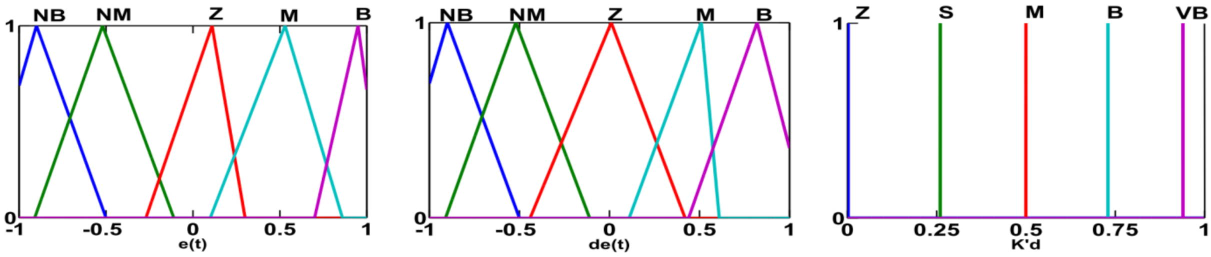

Fig. 8. MFs of the fuzzy D inputs and output after optimization

VI. CONCLUSION

The fuzzy controller characterized by fixed and primary MF rules cannot have good efficiency for systems with high variations and control parameters. In this paper, an autonomous MF tuning method for designing Mamdani fuzzy PID controller is extended Kalman Filter could achieve good speed tracking with respect to different input reference speed in introduced to perform the speed control of DC motors. Results show that the online MF optimization of fuzzy PID controller with comparison with traditional PID and the fuzzy PID controller with fixed MFs.

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