Simulation and Design of a 3-DOF Piezoresistive Accelerometer with Uniform Resolution

Tran Duc Tan

***Abstract*—In this paper, a three-axis piezoresistive accelerometer which has uniform resolutions at all axes was developed using MicroElectroMechanical Systems (MEMS) technology. This sensor is made of a heavy proof mass and four long beams which allow us to obtain high resolutions. By reducing the resonant frequencies, uniform resolutions with small cross axis sensitivity could be designed. This kind of sensor was simulated successfully utilizing a finite element method supported by ANSYS software. The overall chip dimension of the sensor is $1.5 \times 1.5 \times 0.5$ mm^3 and the uniform resolution can be at reached at 0.5 mg.**

***Index Terms*—MEMS, accelerometer, piezoresistance, simulation, uniform resolution.**

I. INTRODUCTION

Micro-machined inertial sensors that consist of accelerometers and gyroscopes have a significant percentage of silicon based sensors. The accelerometer has got the second largest sales volume after pressure sensor [1]. Accelerometer can be found mainly in automotive industry, biomedical application, household electronics, robotics, vibration analysis, navigation system, and so on. Various kinds of accelerometer have increased based on different principles such as capacitive, piezoresistive, piezoelectric, and other sensing ones. The concept of accelerometer is not new but the demand from commerce has motivated continuous researches in this kind of sensor in order to minimize the size and improve its performance.

As we know, the realistic applications create a huge motivation for the widely research of MEMS based sensors, especially accelerometer. In this modern world, applications require new sensors with smaller size and higher performance. In practice, there are rare researches which can bring out an efficient and comprehensive methodology for accelerometer designs.

II. LITERATURE SURVEY AND PROPOSED METHOD

Harkey J.A. et al. [2] presented flicker noise considerations for the design and process optimization of piezoresistive cantilevers. In this work, data was shown which validates the Hooge model for flicker noise in piezoresistive cantilevers. From equations for the Hooge noise, Johnson noise, and sensitivity, an expression was derived to predict force resolution of a piezoresistive cantilever based on its geometry and processing. However, the structure of a cantilever is very simple compared to an accelerometer.

Sankar et al. [3] presents temperature drift analysis of a silicon micro-machined piezoresistive accelerometer. The result is quite simple in terms of the variation of the output voltage at different accelerations and temperatures. The optimization targets on sensitivity or resolution have not mentioned in this work.

C. Pramanik et al. [4] presented the design optimization of high performance conventional silicon-based pressure sensors on flat diaphragms for low-pressure biomedical applications have been achieved by optimizing the doping concentration and the geometry of the piezoresistors. A new figure of merit called the performance factor is defined as the ratio of the product of sensor sensitivity and sensor signal-to-noise ratio to the temperature coefficient of piezoresistance. Performance factor has been introduced as a quantitative index of the overall performance of the pressure sensor for low-range biomedical applications instead of conventional parameters such as sensitivity or resolution.

In the research of Qingzhou Li et al. [5], the authors present the improvement of performance of a monolithic high-shock three-axis piezoresistive accelerometer by using Ant Colony Optimization on the geometric parameters. However, the object of this paper is only the vertical sensitivity. Furthermore, the optimization is based on the combining of Ant Colony Optimization and finite element method software is time consuming.

In the work of Firdaus S.M. et al. [6] studied on a piezoresistive MEMS cantilever which is a mass-based sensor. Author has enhanced the sensor's sensitivity by utilizing the stress concentration region. The resolution improvement has not mentioned yet.

In 2010, Prasenjit Ray et al. [7] proposed an optimum design which is studied for the maximum bending stress of an acceleration. It is based on a new design of SU8 based piezoresistive accelerometer, where SU8/carbon black is used as piezoresistors. The accelerometer structure is optimized to generate maximum stress on the surface of the beams. However, the fabrication process is more complex than usual.

We have successfully simulated, designed and fabricated a 3-DOF piezoresistive accelerometer [13]. This paper is an extended research of that work when uniform resolutions sensor is required for specific applications such as robotics, biomedical application, etc. To obtain good resolutions, we have to maximize the sensitivity and minimize the noise. They both can be solved by reducing the resonant frequency. However, we could not overcome the limitation of practice

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fabrication (the beam structure is very easy to be cracked while increasing the beam's length or reducing the beam's thickness and width). Thus, we first optimized the sensitivity of the sensor by suitable placing the piezoresistor on the surface of the four beams. After that, we could reduce the noise by narrowing the bandwidth at different values by using analogue low pass filter in the readout circuit to achieve uniform resolutions.

III. PROPOSED 3-DOF ACCELEROMETER WITH UNIFORM AXIAL RESOLUTIONS

The three-degrees-of-freedom accelerometer always requires small cross-axial acceleration, high and linear sensitivity, and small resolutions. We proposed a flexure configuration that is shown in Fig. 1 in order to meet these critical characteristics [12].

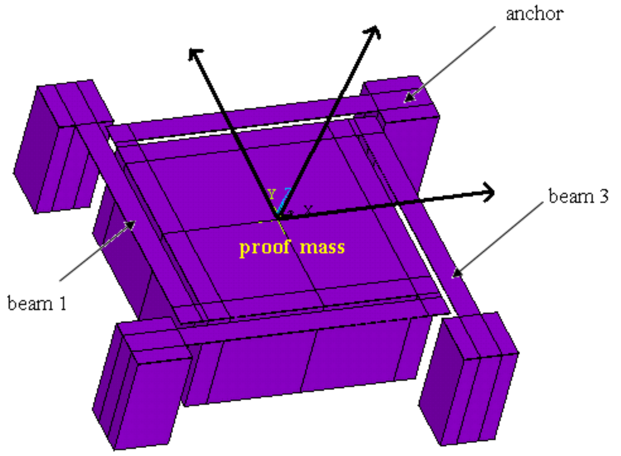


Fig. 1. 3D model of the 3-DOF piezoresistive accelerometer

The overall chip dimension is 1.5×1.5×0.5 mm^3 (L×W×T). Twelve piezoresistors are diffused on the surface of beam structure. Three simple Wheatstone bridges are formed directly on this sensor by interconnecting these piezoresistors to sense three components of acceleration independently.

The piezoresistance effect is known to be caused by the anisotropic characteristics of the energy resolution in crystal space [8]. The longitudinal piezoresistance coefficient π_l and transverse piezoresistance coefficient π_t in directions <110> and <1$\bar{1}$0> of n-type silicon (100) can be expressed as [9]:

$$\begin{aligned} \pi_l &= \frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44}) \\ \pi_t &= \frac{1}{2}(\pi_{11} + \pi_{12} - \pi_{44}) \end{aligned} \tag{1}$$

where π_{11} , π_{12} and π_{44} are only three independent piezoresistive coefficients.

From simulation results [13], we found that two normal stresses are rather smaller when comparing to the longitudinal stress σ_l . Thus, we can calculate the relative change of resistance due to the normal stress by the following equation:

$$\frac{\Delta R}{R} \approx \pi_l \sigma_l \tag{2}$$

The mechanical sensitivities of each components of acceleration can be respectively expressed as:

$$S^i_{stress} = \frac{\sigma^i}{a_i} \qquad i = X, Y, Z \tag{3}$$

where S^i_{stress} is the mechanical sensitivity and σ^i is longitudinal stress induced by the acceleration i^{th} component a_i . The electronics sensitivity can be given by:

$$S_i = \frac{V_{out}}{a_i} = \frac{\Delta R}{R} V_{in} = \pi_l S^i_{stress} V_{in} \tag{4}$$

where S_i and V_{out} are the sensitivity to the ith acceleration component and output voltage, respectively. The longitudinal stress σ^i in Equ. (3) obtained from the stress analysis by utilizing ANSYS software. This value is stress at the center point of piezoresistors and on the surface of the beam.

From equation (4), it is obvious that the sensitivities in three axes can be optimized if the twelve piezoresistors are diffused at specific locations that can lead to the maximize longitudinal stresses σ^i .

The proof mass *m* moves from its neutral position relative to the frame when the frame starts to accelerate. For a given acceleration *a*, the proof mass displacement *x* is determined by the mechanical suspension *k* and the damping *b*:

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = a(t) \tag{5}$$

This equation can be rewritten:

$$\frac{d^2x}{dt^2} + 2\xi\omega_n \frac{dx}{dt} + \omega_n^2 x = a(t) \tag{6}$$

where $\omega_n = \sqrt{\frac{k}{m}}$ is natural resonant frequency, and $\xi = \frac{b}{2m\omega_n}$ is damping factor.

The transfer function can be obtained as:

$$\frac{X(s)}{A(s)} = \frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2} \tag{7}$$

It can be seen that an open loop accelerometer is equivalent to a "*mechanical*" low pass filter. The natural resonant frequency is an important parameter in an open loop accelerometer design. It is designed to satisfy the requirements on the sensitivity and the bandwidth.

Resolution is defined as the noise divided by the sensitivity. It is observed that optimization of the resolution has been achieved by increasing the sensitivity and reducing the noise.

There are two typical noise sources existing in all piezoresistive sensors, including the Johnson noise and flicker noise [10, 11]. The noises depend on the bandwidth of sensor, the temperature, the geometry of piezoresistor, the doping concentration and also the thickness of the beam.

Johnson noise (thermal noise) is the electronic noise generated by the thermal agitation of the charge carriers inside an electrical conductor when applying an arbitrary voltage. The power spectral density (PSD) of thermal noise is nearly constant throughout the frequency spectrum. It means that Johnson noise can be assumed to be White noise.

In fact, it is hardly to observe this noise in a realistic accelerometer because electrical noise in the measurement circuit is often larger. The root mean square voltage of equivalent acceleration noise in each piezoresistor is:

$$V_i^{Johnson} = \sqrt{4k_B T B_i R} \quad i = X, Y, Z \tag{8}$$

where k_B= 1.38 x 10^{-23} J/K is Boltzmann's constant, T is temperature in resistors, R is resistance value of the piezoresistor, and B is measured bandwidth. The bandwidth can be determined by many parameters such as the sampling frequency, analogue filtering, the resonant frequency of the mechanical structure, or losses in the wires, etc.

Resolution is defined as the noise divided by the sensitivity:

$$R_i = \frac{V_i^{noise}}{S_i} \qquad i = X, Y, Z \tag{9}$$

It can be seen that the resolution can be minimized by reducing noise and maximizing the sensitivity. The sensitivity can not quite large due to the fabrication constraints. However, we can reduce the noise by constraint the bandwidth of the accelerometer by integrating a low pass filter at the circuit interface which has a two order transform function as:

$$C(s) = \frac{1}{s^2 + 2\xi_c \omega_{cn} s + \omega_{cn}^2} \tag{10}$$

where $a_1 = 2\xi_c \omega_{cn}$ and $a_2 = \omega_{cn}^2$ are two coefficients of the analogue low pass filter. We can adjust the constraint bandwidth by using a variable resistor to adjust the a_1 and a_2.

The combined system has the transform function is:

$$H(s) = \frac{1}{s^2 + 2\xi \omega_n s + \omega_n^2} \frac{1}{s^2 + 2\xi_c \omega_{cn} s + \omega_{cn}^2} \tag{11}$$

Fig. 2 shows the illustration of using low pass filter to reduce the bandwidth Bi of the accelerometer. From the equations (8) and (9), we can see that the resolution Ri can be three times improved. Note that this method does not change the sensitivity or structure of sensor; thus, ensure the stability of the accelerometer.

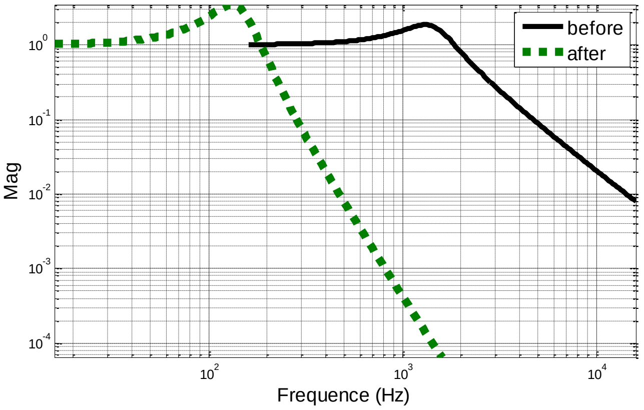


Fig. 2. Frequency responses of the accelerometer before and after constrain bandwidth.

IV. DESIGN AND SIMULATION USING ANSYS

The most important aspect of our design process which requires finite element method (FEM) is the analysis of the stress distribution in the flexure beams. Based on this distribution, piezoresistors are positioned to eliminate the cross-axis sensitivities and to maximize the sensitivities to the three acceleration components. The finite element model of the sensing chip was analysed by using ANSYS software.

Fig. 3 shows the Von Mise stress analysis when the a_x acceleration is applied to the sensor.

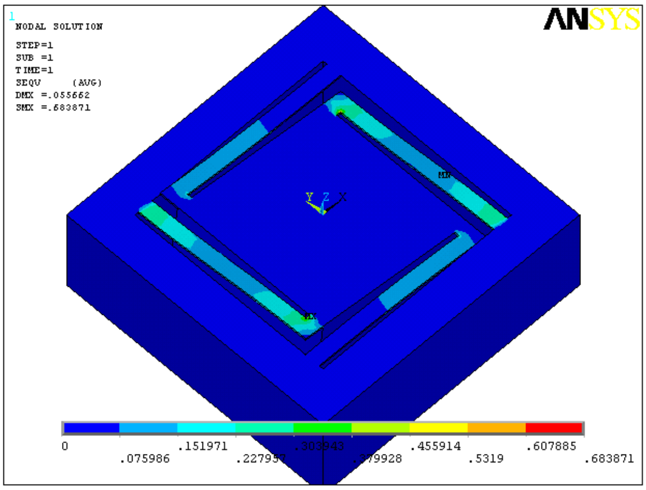
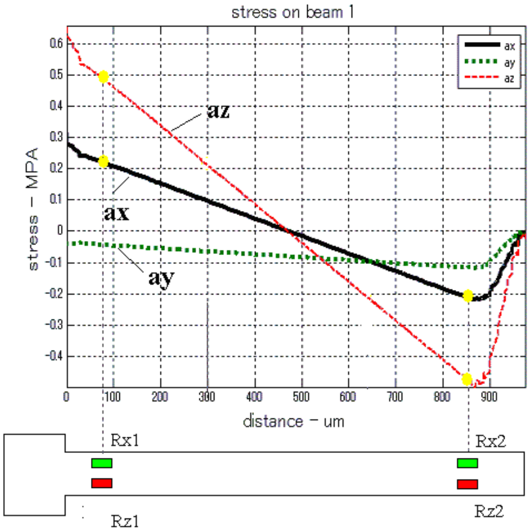
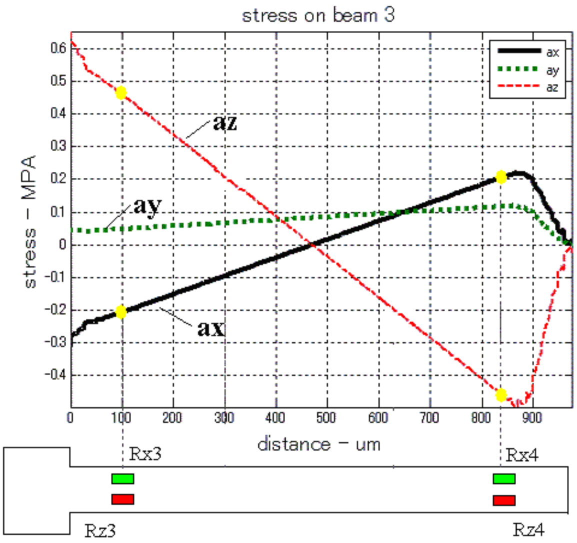


Fig. 3. The stress distribution on the beams caused by the acceleration a_x.



(a)



(b)

Fig. 4. Longitudinal stresses on the surface of the 1st (4.a) and the 3rd (4.b) beams due to the 1 *g* acceleration.

Fig. 4 shows the longitude stress along the 1st and the 3rd beams when the sensor is submitted to acceleration in three directions (X, Y and Z). From this figure, we can point out the optimal locations to diffuse the piezoresistors in order to sense accelerations a_x and a_z with very small cross-talk. To get the maximum stress corresponding to three cases of applying acceleration, the locations of piezoresistors are also illustrated in this figure. Similarly, a_y acceleration can be sensed via four piezoresistors on the 2nd and the 4th beams.

Table 1 summarizes the increase (+), decrease (-), or invariance (0) in resistance of piezoresistors due to application of accelerations a_x, a_y, and a_z. These identical piezoresistors are diffused on the surface of the beams to form three Wheatstone bridges [10]. Three Wheatstone bridge circuits are then connected to three corresponding low pass filters that have been discussed in the section 2.

TABLE I: RESISTANCE VALUES CHANGES WITH THREE COMPONENTS OF ACCELERATION

	Rz_1	Rz_2	Rz_3	Rz_4	Ry_1	Ry_2	Ry_3	Ry_4	Rx_1	Rx_2	Rx_3	Rx_4
a_z	+	-	+	-	-	-	-	-	-	-	-	-
a_y	-	0	+	0	-	+	-	+	0	+	+	0
a_x	-	-	+	+	0	+	+	0	-	+	-	+

The performance of the accelerometer can be summarized in Table 2. To achieve the uniform resolutions, we have reduced 5.5 times the bandwidths of X and Y acceleration components.

TABLE II: RESISTANCE VALUES CHANGES WITH THREE COMPONENTS OF ACCELERATION

	Sensitivity (mV/V/g)	Johnson noise per 1 piezoresistor (μV) before improvements	Johnson noise per 1 piezoresistor (μV) after improvements	Resolution (mg)
a_z	0.336	0.415	0.415	0.5
a_y a_x	0.152	0.513	0.19	

V. CONCLUSION

This paper presents a design and simulation of 3-DOF MEMS based accelerometer with uniform resolution. The piezoresistive effect was used as sensing principle of the sensor. The most important aspect of Finite Element Analysis in our design process is the analysis of the stress distribution in the four flexure beams. The stress analysis was performed in order to determine the positions of the doped piezoresistors on these beams. The miniature 3-DOF accelerometer with uniform resolution is expected to be applied in various kinds of applications such as biomedical, robotics, navigation, etc.

ACKNOWLEDGMENT

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