

# New Analysis on $H_\infty$ Control for Exponential Stability of Artificial Neural Network with Mixed Time-Varying Delays via Hybrid Feedback Control

C. Chantawat, T. Botmart, and W. Weera

**Abstract**—This paper is concerned the problem of  $H_\infty$  control for artificial neural networks with mixed time-varying delays which comprising different interval and distributed time-varying delays via hybrid feedback control. The interval and distributed time-varying delays are not necessary to be differentiable. The main purpose of this research is to estimate exponential stability of artificial neural network with  $H_\infty$  performance attenuation level  $\gamma$ . The key features of the approach include the introduction of a new Lyapunov-Krasovskii functional with triple integral terms, the employment of a tighter bounding technique, some slack matrices and newly introduced convex combination condition in the calculation, improved delay-dependent sufficient conditions for the  $H_\infty$  control with exponential stability of the system are obtained in terms of linear matrix inequalities (LMIs). The results of this paper complement the previously known ones. Finally, a numerical example is presented to show the effectiveness of the proposed methods.

**Index Terms**—Artificial neural networks, exponential stability,  $H_\infty$  control, hybrid feedback control.

## I. INTRODUCTION

During the past decades, the problem of the reliable control has received much attention [1]-[4]. Neural networks have received considerable due to the effective use of many aspects such as signal processing, automatic control engineering, associative memories, parallel computation, fault diagnosis, combinatorial optimization and pattern recognition and so on [5]-[7]. It has been shown that the presence of time delay in a dynamical system is often a primary source of instability and performance degradation [8]. Many researchers have paid attentions to the problem of stability for systems with time delays [9], [10]. The  $H_\infty$  controller can be used to guarantee closed loop system not only a stability but also an adequate level of performance. In practical control systems, actuator faults, sensor faults or some component faults may happen, which often lead to

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unsatisfactory performance, even loss of stability. Therefore, research on reliable control is necessary.

Most of the works have been focused on the problem of designing a  $H_\infty$  controller that stabilizes linear systems with time-varying. Also, it is assumed that the perfect information is available for state feedback and the controlled output is disturbance free. The problem of  $H_\infty$  control design usually leads to solving an algebraic Lyapunov equation. It should be noted that some works have been dedicated to the problem of reliable control for nonlinear systems with time-varying delay [2]-[4]. However, to the best of the authors' knowledge, so far the research on reliable  $H_\infty$  control is still an open problem, which is worth further investigation.

Motivated by above discussion, in this paper we have considered the problem of  $H_\infty$  control for neural networks with mixed time-varying delays which comprising different interval and distributed time-varying delays via hybrid feedback control. The interval and distributed time-varying delays are not necessary to be differentiable. The main purpose of this research is to estimate exponential stability of neural network with  $H_\infty$  performance attenuation level  $\gamma$ . The key features of the approach include the introduction of a new Lyapunov-Krasovskii functional with triple integral terms, the employment of a tighter bounding technique, some slack matrices and newly introduced convex combination condition in the calculation, improved delay-dependent sufficient conditions for the  $H_\infty$  control with exponential stability of the system are obtained in terms of linear matrix inequalities (LMIs). The results of this paper complement the previously known ones. Finally, a numerical example is presented to show the effectiveness of the proposed methods.

The rest of this paper is organized as follows. In Section II, some notations, definitions and some well-known technical lemmas are given. Section III presents the  $H_\infty$  control for exponential stability and the  $H_\infty$  control for exponential stability. The numerical examples and their computer simulations are provided in Section IV to indicate the effectiveness of the proposed criteria. Finally, the paper is concluded in Section V.

## II. DEFINITIONS OF FUNCTION SPACES AND NOTATION

Consider the following artificial neural network system with mixed time delays

$$\begin{aligned} \dot{x}(t) &= -Ax(t) + Bf(x(t)) + Cg(x(t-h(t))) \\ &\quad + D \int_{t-d(t)}^t h(x(s))ds + Ew(t) + U(t), \\ z(t) &= A_1x(t) + B_4x(t-h(t)) + C_1u(t) \\ &\quad + D_1 \int_{t-d(t)}^t x(s)ds + E_1w(t), \\ x(t) &= \phi(t), t \in [-\rho, 0], \end{aligned} \quad (1)$$

where  $x(t) \in R^n$  is the state vector,  $w(t) \in R^n$  the deterministic disturbance input,  $z(t) \in R^n$  the system output,  $f(x(t)), g(x(t)), h(x(t))$  the neuron activation function,  $A = \text{diag}\{a_1, \dots, a_n\} > 0$  a diagonal matrix,  $B, C, D, E, A_1, B_4, C_1, D_1, E_1$  are the known real constant matrices with appropriate dimensions,  $\phi(t) = C[[-\rho, 0], R^n]$  the initial function, The state hybrid feedback controller  $U(t)$  satisfies:

$$U(t) = B_1u(t) + B_2u(t-\tau(t)) + B_3 \int_{t-d_1(t)}^t u(s)ds, \quad (2)$$

where  $u(t) = Kx(t)$  and  $K$  is a constant matrix control gain,  $B_1, B_2, B_3$  are the known real constant matrices with appropriate dimensions. Then, substituting it into (1), it is easy to get the following:

$$\begin{aligned} \dot{x}(t) &= [-A + B_1K]x(t) + Bf(x(t)) + Ew(t) \\ &\quad + Cg(x(t-h(t))) + D \int_{t-d(t)}^t h(x(s))ds \\ &\quad + B_2Kx(t-\tau(t)) + B_3K \int_{t-d_1(t)}^t x(s)ds, \\ z(t) &= [A_1 + C_1K]x(t) + B_4x(t-h(t)) \\ &\quad + D_1 \int_{t-d(t)}^t x(s)ds + E_1w(t), \\ x(t) &= \phi(t), t \in [-\rho, 0], \end{aligned} \quad (3)$$

where the time-varying delays function  $h(t), \tau(t), d(t)$  and  $d_1(t)$  satisfy the condition;

$$\begin{aligned} 0 \leq h_1 \leq h(t) \leq h_2, \quad 0 \leq d(t) \leq d, \\ 0 \leq \tau(t) \leq \tau, \quad 0 \leq d_1(t) \leq d_1, \end{aligned} \quad (4)$$

where  $h_1, h_2, \tau, d, d_1, \rho = \max\{h_2, \tau, d, d_1\}$  are known real constant scalars and we denote  $h_{12} = h_2 - h_1$ .

In this paper, we consider activation functions  $f(\cdot), g(\cdot),$  and  $h(\cdot)$  satisfy Lipschitzian with the Lipschitz constants  $\hat{f}_i, \hat{g}_i$  and  $\hat{h}_i > 0$ :

$$\begin{aligned} |f_i(x_1) - f_i(x_2)| &\leq \hat{f}_i |x_1 - x_2|, \\ |g_i(x_1) - g_i(x_2)| &\leq \hat{g}_i |x_1 - x_2|, \\ |h_i(x_1) - h_i(x_2)| &\leq \hat{h}_i |x_1 - x_2|, \end{aligned} \quad (5)$$

and we denote that

$$\begin{aligned} F &= \text{diag}\{\hat{f}_i, i=1, 2, \dots, n\}, G = \text{diag}\{\hat{g}_i, i=1, 2, \dots, n\}, \\ H &= \text{diag}\{\hat{h}_i, i=1, 2, \dots, n\}, \end{aligned} \quad (6)$$

**Definition 1.** [11]. Given  $\alpha > 0$ . The zero solution of system (1), where  $u(t) = 0, w(t) = 0$ , is  $\alpha$ -stable if there is a positive number  $N > 0$  such that every solution of the system satisfies

$$\|x(t, \phi)\| \leq N \|\phi\|_c e^{-\alpha t}, \quad \forall t \geq 0.$$

**Definition 2.** [11]. Given  $\alpha > 0, \gamma > 0$ . The  $H_\infty$  control problem for system (1) has a solution if there exists a memoryless state feedback controller  $u(t) = Kx(t)$  satisfying the following two requirements:

(i) The zero solution of the closed-loop system, where

$$w(t) = 0,$$

$$\begin{aligned} \dot{x}(t) &= -Ax(t) + Bf(x(t)) + Cg(x(t-h(t))) \\ &\quad + D \int_{t-d(t)}^t h(x(s))ds + U(t), \end{aligned}$$

is  $\alpha$ -stable.

(ii) There is a number  $c_0 > 0$  such that

$$\sup \frac{\int_0^\infty \|z(t)\|^2 dt}{c_0 \|\phi\|_c^2 + \int_0^\infty \|w(t)\|^2 dt} \leq \gamma,$$

where the supremum is taken over all  $\phi(\cdot) \in C[[-\rho, 0], R^n]$  and the non-zero uncertainty  $w(\cdot) \in L_2([0, \infty), R^n)$ .

**Lemma 3.** [12]. (Cauchy inequality) For any symmetric positive definite matrix  $N \in R^{n \times n}$  and  $x, y \in R^n$  we have

$$\pm 2x^T y \leq x^T N x + y^T N^{-1} y.$$

**Lemma 4.** [12]. Given a positive definite matrix  $Z \in R^{n \times n}$ , and two scalar  $0 \leq r_1 < r_2$  and vector function  $x: [r_1, r_2] \rightarrow R^n$  such that the following integrations concerned are well defined, then we have

$$\left( \int_{r_1}^{r_2} x(s)ds \right)^T Z \left( \int_{r_1}^{r_2} x(s)ds \right) \leq (r_2 - r_1) \int_{r_1}^{r_2} x^T(s) Z x(s)ds.$$

**Lemma 5.** [13]. For any positive definite symmetric constant matrix  $P$  and scalar  $\tau > 0$ , such that the following integrations are well defined

$$\begin{aligned} - \int_{-\tau}^0 \int_{t+\theta}^t x^T(s) P x(s) ds d\theta \\ \leq - \frac{2}{\tau^2} \left( \int_{-\tau}^0 \int_t x(s) ds d\theta \right)^T P \left( \int_{-\tau}^0 \int_t x(s) ds d\theta \right). \end{aligned}$$

**Lemma 6.** [12]. (Schur complement) Given constant matrices  $Z, Z_1, Z_2, Z_3$  where  $Z = Z^T$  and  $Z_i = Z_i^T > 0$

Then  $Z + Z_1^T Z^{-1} Z_1 < 0$  if and only if

$$\begin{pmatrix} Z & Z^T \\ Z^T & -Z \end{pmatrix} < 0 \quad \text{or} \quad \begin{pmatrix} -Z & Z \\ Z^T & Z \end{pmatrix} < 0.$$

## III. STABILITY ANALYSIS

In this section, we will present stability criterion for system (3). Consider a Lyapunov-Krasovskii functional candidate as

$$V(t, x_t) = \sum_{i=1}^{14} V_i(t, x_t), \quad (7)$$

where

$$\begin{aligned} V_1(x) &= x^T(t) P x(t) + 2x^T(t) P \int_{t-h}^t x(s) ds \\ &\quad + \left( \int_{t-h}^t x(s) ds \right)^T P_3 \int_{t-h}^t x(s) ds + 2x^T(t) P_4 \\ &\quad \times \int_{-h_2}^0 \int_{t+s}^t x(\theta) d\theta ds + 2 \left( \int_{t-h_2}^t x(s) ds \right)^T P_5 \\ &\quad \times \int_{-h_2}^0 \int_{t+s}^t x(\theta) d\theta ds + \left( \int_{-h_2}^0 \int_{t+s}^t x(\theta) d\theta ds \right)^T \\ &\quad \times P_6 \int_{-h_2}^0 \int_{t+s}^t x(\theta) d\theta ds \\ V_2(x) &= \int_{t-h_1}^t e^{2\alpha(s-t)} x^T(s) R x(s) ds, \\ V_3(x_t) &= \int_{t-h_1}^t e^{2\alpha(s-t)} x^T(s) R_2 x(s) ds, \\ V_4(x) &= h \int_{t-h_1}^0 \int_{t+s}^t e^{2\alpha(\theta-t)} x^T(\theta) Q x(\theta) d\theta ds, \\ V_5(x) &= h \int_{t-h_1}^0 \int_{t+s}^t e^{2\alpha(\theta-t)} \dot{x}^T(\theta) Q \dot{x}(\theta) d\theta ds, \\ V_6(x) &= h \int_{-h_1}^{-h_2} \int_{t+s}^t e^{2\alpha(\theta-t)} x^T(\theta) Z x(\theta) d\theta ds, \\ V_7(x) &= \int_{-d}^0 \int_{t+s}^t e^{2\alpha(\theta-t)} h^T(x(\theta)) U h(x(\theta)) d\theta ds, \\ V_8(x) &= \int_{-d_1}^0 \int_{t+s}^t e^{2\alpha(\theta-t)} u^T(\theta) S u(\theta) d\theta ds, \\ V_9(x) &= \tau \int_{-d_1}^0 \int_{t+s}^t e^{2\alpha(\theta-t)} u^T(\theta) S u(\theta) d\theta ds, \\ V_{10}(x_t) &= \int_{-h}^{-\tau} \int_{t+s}^t \int_{t+u}^s e^{2\alpha(\theta+t-u)} x^T(\theta) Z_1 x(\theta) d\theta du ds, \\ V_{11}(x) &= \int_{-h_1}^0 \int_{t+s}^t e^{2\alpha(\theta+s-t)} x^T(\theta) W x(\theta) d\theta ds d\tau, \\ V_{12}(x) &= \int_{-h}^0 \int_{t+s}^t e^{2\alpha(\theta+s-t)} x^T(\theta) W x(\theta) d\theta ds d\tau, \\ V_{13}(x_t) &= \int_{-h}^0 \int_{t+s}^t \int_{t+u}^s e^{2\alpha(\theta+s-t)} x^T(\theta) W_3 x(\theta) d\theta ds d\tau, \\ V_{14}(x) &= \int_{-d}^0 \int_{t+s}^t e^{2\alpha(\theta-t)} x^T(\theta) Q x(\theta) d\theta ds, \end{aligned}$$

Let us set

$$\begin{aligned} \lambda &= \lambda(P) \\ \lambda_2 &= 3h \lambda_{\max}(R_1) + h^2 \lambda_{\max}(Q_1) + h^2 \lambda_{\max}(Z_2) \\ &\quad + d^2 \lambda_{\max}(H^T U H) + d^2 \lambda_{\max}(P^{-1} B^T S B P^{-1}) \\ &\quad + \tau^2 \lambda_{\max}(P^{-1} B^T S B P^{-1}) + h^2 \lambda_{\max}(W) \\ &\quad + h^2 \lambda_{\max}(W) + d^2 \lambda_{\max}(Q) + h \lambda_{\max}(Z) \end{aligned}$$

**Theorem 7.** Given  $\alpha > 0$ , The  $H_\infty$  control of system (3) has a solution if there exist symmetric positive definite matrices  $Q_1, Q_2, Q_3, R_1, R_2, S_1, S_2, S_3, W_1, W_2, W_3, Z_1, Z_2, Z_3$ , diagonal matrices  $U > 0, U_2 > 0, U_3 > 0$ , and matrices

$P_1 = P_1^T, P_3 = P_3^T, P_6 = P_6^T, P_2, P_4, P_5$  such that the following LMI hold:

$$\Sigma_1 = \begin{bmatrix} \Sigma_{1(1,1)} & \Sigma_{1(1,2)} \\ * & \tilde{\Sigma}_{1(2,2)} \end{bmatrix} < 0, \quad (8)$$

$$\Sigma_2 = \begin{bmatrix} \tilde{\Sigma}_{2(1,1)} & \tilde{\Sigma}_{2(1,2)} \\ * & \tilde{\Sigma}_{2(2,2)} \end{bmatrix} < 0, \quad (9)$$

$$\Sigma_3 = \begin{bmatrix} -0.5e^{-2\alpha h_2} Q_2 + N \end{bmatrix} < 0, \quad (10)$$

$$\Sigma_4 = \begin{bmatrix} -0.1R_1 + \tau^2 B_1^T S_1 B_1 \end{bmatrix} < 0, \quad (11)$$

where

$$\begin{aligned} \Sigma_{1(1,1)} &= \begin{bmatrix} \Pi & F^T P & P & 2dPD \\ * & -U_2 & -U & \theta \\ * & * & * & \Sigma \\ * & * & * & \Sigma \end{bmatrix}, \\ \Sigma_{1(1,2)} &= \begin{bmatrix} 4PB_{1,2} & 2dPB_{1,3} & PE \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ \tilde{\Sigma}_{1(2,2)} &= \begin{bmatrix} \Sigma_{1(5,5)} & \Sigma^0 & \theta \\ * & * & * \\ * & * & -0.5\gamma \end{bmatrix}, \\ \Sigma_{2(1,1)} &= \begin{bmatrix} -0.4R_1 & R_1 B & R_1 C & 2dR_1 D \\ * & -U_2 & -U_3 & \theta \\ * & * & * & \Sigma_{2(4,4)} \end{bmatrix}, \\ \Sigma_{2(1,2)} &= \begin{bmatrix} 4R_0 B_2 & 2dR_0 B_3 & R_0 B_1 & R_0 E \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ \Sigma_{2(2,2)} &= \begin{bmatrix} \Sigma_{2(5,5)} & \Sigma^0 & \theta & \theta \\ * & * & * & * \\ * & * & -S_3 & * \\ * & * & * & -0.5\gamma \end{bmatrix}, \\ \Pi &= \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} < 0, \\ \Pi_{11} &= \begin{bmatrix} \Pi_{1,1} & \Pi_{1,2} & \Pi_{1,3} & \Pi_{1,4} & \Pi_{1,5} \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & \Pi_{5,5} \end{bmatrix} \end{aligned}$$

$$\Pi_{12} = \begin{bmatrix} \Pi_{1,6} & \Pi_{1,7} & 0 & \Pi_{1,9} & -A^T R_1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -P_3 & 0 & 0 & -P_5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_{22} = \begin{bmatrix} \Pi_{2,6} & 0 & 0 & \Pi_{2,9} & P_2 \\ * & \Pi & 0 & 0 & 0 \\ * & * & \Pi_{8,8} & 0 & 0 \\ * & * & * & -2\alpha P_6 & P_4 \\ * & * & * & * & \Pi_{10,10} \end{bmatrix},$$

$$\begin{aligned} \Pi_{1,1} = & -AP_1 - P_1A^T + B_1B_1 + B_1^T B_1^T + R_1 + P_2 + P_2^T + R_2 \\ & + B^T U B + h P - e^{-2\alpha h} Q - 0.5e^{-2\alpha h_2} Q + dQ \\ & + dH^T UH - e^{-4\alpha h} (W + W^T) - e^{-4\alpha h} (Z + Z^T) \\ & - e^{-4\alpha h} (W_2 + W_2^T + W_3 + W_3^T) + h P^T - 2\alpha P_1 \\ & + F^T U F, \end{aligned}$$

$$\Pi = e^{-2\alpha h} Q, \quad \Pi = -P + e^{-2\alpha h_2} Q,$$

$$\begin{aligned} \Pi_{1,5} = & 2h_1^{-1} e^{-2\alpha h} W_1, \quad \Pi_{1,6} = P_3 - P_4 + h_2 P_5^T - 2\alpha P_2 \\ & + 2h_2^{-1} e^{-4\alpha h_2} (W_2 + W_3), \quad \Pi_{1,7} = 2h_{12}^{-1} e^{-4\alpha h_2} Z_1, \end{aligned}$$

$$\begin{aligned} \Pi_{1,9} = & P_5 + h_2 P_6 - 2\alpha P_4, \quad \Pi_{2,2} = -e^{-2\alpha h_1} (R_1 - Q_1) \\ & - e^{-2\alpha h_2} Z_2, \quad \Pi_{2,3} = e^{-2\alpha h_2} (Z_2 - Z_3), \end{aligned}$$

$$\begin{aligned} \Pi_{2,4} = & e^{-2\alpha h} Z, \quad \Pi_{2,5} = -e^{-2\alpha h} (Z + Z^T + Z_2 + Z_2^T) \\ & + G^T C^T U_3 CG + G^T U_3 G, \end{aligned}$$

$$\Pi_{3,4} = e^{-2\alpha h_2} (Z_2 - Z_3), \quad \Pi_{4,4} = -e^{-2\alpha h_2} (Z_2 + R_2 + Q_2),$$

$$\begin{aligned} \Pi_{5,5} = & -h_1^{-2} e^{-4\alpha h} (W_1 + W_1^T), \quad \Pi_{6,6} = -P_5 - P_5^T - 2\alpha P_3 \\ & - h_2^{-2} e^{-4\alpha h_2} (W_2 + W_2^T + W_3 + W_3^T), \end{aligned}$$

$$\Pi_{7,7} = -h^{-2} e^{-4\alpha h_2} (Z + Z^T), \quad \Pi_{8,8} = -d^{-1} e^{-2\alpha d} Q_3,$$

$$\begin{aligned} \Pi_{10,10} = & -1.5R_1 + h_2^2 Q_1 + h_2^2 Q_2 + h_{12}^2 Z_2 + h_{12} h Z_1 \\ & + hW_1 + hW_2 + hW_3, \end{aligned}$$

$$\sum_{1(4,4)} = -2de^{-2\alpha d} U, \quad \sum_{1(5,5)} = -4e^{-2\alpha t} S_1,$$

$$\sum_{1(6,6)} = -2d e^{-2\alpha d_1} S_2, \quad \sum_{2(4,4)} = -2de^{-2\alpha d} U,$$

$$\sum_{2(5,5)} = -4e^{-2\alpha t} S_1, \quad \sum_{2(6,6)} = -2d_1 e^{-2\alpha d_1} S_2,$$

$$N = e^{-2\alpha t} B_1^T S_1 B_1 + dB_1^T S_2 B_1 + B_1^T S_3 B_1$$

Moreover, stabilizing feedback control is given by

$$u(t) = B_1 P_1^{-1} x(t), \quad t \geq 0,$$

and the solution of the system satisfies

$$\|x(t, \phi)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} \|\phi\| e^{-\alpha t}, \quad t \geq 0.$$

**Proof:** Choosing the Lyapunov-Krasovskii functional candidate as (7). It is easy to check that

$$\lambda_1 \|x(t)\|^2 \leq V(t, x_t), \quad \forall t \geq 0, \quad (12)$$

$$\text{and } V(0, x_0) \leq \lambda_2 \|\phi\|_c^2.$$

We take the time-derivative of  $V_i(x_t)$  along the solutions of system (3)

$$\begin{aligned} \dot{V}_1(x_t) = & -2x^T(t)AP_1x(t) + 2x^T(t)B_1^T B_1x(t) \\ & + 2f^T(x(t))B^T P_1x(t) + 2w^T(t)E^T P_1x(t) \\ & + 2u^T(t - \tau(t))B^T P_1x(t) + 2g^T(x(t - h(t)))C^T P_1x(t) \\ & + 2\left(\int_{t-d(t)}^t h(x(s))ds\right)^T D^T P_1x(t) \end{aligned}$$

$$\begin{aligned} & + 2\left(\int_{t-d_1(t)}^t u(s)ds\right)^T B_1^T P_1x(t) + 2x^T(t)P_2x(t) \\ & - 2x^T(t)P_3x(t-h) + 2\left(\int_{t-h}^t x(s)ds\right)^T P_2x(t) \\ & + 2[x(t) - x(t-h)]^T P_3 \int_{t-h}^t x(s)ds \end{aligned}$$

$$\begin{aligned} & + 2x^T(t)P_4 \left[ h_2x(t) - \int_{t-h_2}^t x(s)ds \right] \\ & + 2\int_{-h_2}^0 \int_{t+s}^t x(\theta)d\theta ds \left[ P_4x(t) \right] \end{aligned}$$

$$\begin{aligned} & + 2\left(\int_{t-h_2}^t x(s)ds\right)^T P_5 \left[ h_2x(t) - \int_{t-h_2}^t x(s)ds \right] \\ & + 2[x(t) - x(t-h_2)]^T P_5 \left[ \int_{t-h_2}^t \int_{t+s}^t x(\theta)d\theta ds \right] \end{aligned}$$

$$\begin{aligned} & + 2[h_2x(t) - x(t-h)]^T P_6 \left( \int_{t-h}^t \int_{t+s}^t x(\theta)d\theta ds \right) \end{aligned}$$

$$\dot{V}_2(x_t) = x^T(t)R_1x(t) - e^{-2\alpha h_1} x^T(t-h_1)R_1x(t-h_1) - 2\alpha V_2,$$

$$\dot{V}_3(x_t) = x^T(t)R_2x(t) - e^{-2\alpha h_2} x^T(t-h_2)R_2x(t-h_2) - 2\alpha V_3,$$

$$\begin{aligned} \dot{V}_4(x_t) \leq & h_1^2 \dot{x}^T(t)Q_1\dot{x}(t) - h_1 e^{-2\alpha h_1} \int_{t-h_1}^t \dot{x}^T(s)Q_1\dot{x}(s)ds \\ & - 2\alpha V_4, \end{aligned}$$

$$\begin{aligned} \dot{V}_5(x_t) \leq & h_2^2 \dot{x}^T(t)Q_2\dot{x}(t) - h_2 e^{-2\alpha h_2} \int_{t-h_2}^t \dot{x}^T(s)Q_2\dot{x}(s)ds \\ & - 2\alpha V_5, \end{aligned}$$

$$\dot{V}_6(x_t) \leq h_{12}^2 \dot{x}^T(t)Z_2\dot{x}(t) - h_{12} e^{-2\alpha h_{12}} \int_{t-h_{12}}^{t-h_1} \dot{x}^T(s)Z_2\dot{x}(s)ds$$

$$\begin{aligned} \dot{V}_7(x_t) \leq & dh^T(\dot{x}(t))Uh(x(t)) - e^{-2\alpha d} \int_{t-d}^t h^T(x(s))Uh(x(s))ds \\ & - 2\alpha V_7, \end{aligned}$$

$$\begin{aligned} \dot{V}_8(x_t) \leq & d u^T(t)S_1 u(t) - e^{-2\alpha d_1} \int_{t-d_1}^t u^T(s)S_1 u(s)ds - 2\alpha V_8, \end{aligned}$$

$$\dot{V}_9(x_t) \leq \tau^2 \dot{u}^T(t)S_1 \dot{u}(t) - \tau e^{-2\alpha \tau} \int_{t-\tau}^t \dot{u}^T(s)S_1 \dot{u}(s)ds - 2\alpha V_9,$$

$$\begin{aligned} \dot{V}_{10}(x_t) \leq & h_{12} h_2 \dot{x}^T(t)Z_1\dot{x}(t) - e^{-4\alpha h_2} \int_{-h_2}^{-h_1} \int_{t+\theta}^t u^T(s)S_2 u(s)ds d\theta \\ & - 2\alpha V_{10}, \end{aligned}$$

$$\begin{aligned} \dot{V}_{11}(x_t) \leq & h_1 \dot{x}^T(t)W_1\dot{x}(t) - e^{-4\alpha h_1} \int_{-h_1}^0 \int_{t+\tau}^t \dot{x}^T(s)W_1\dot{x}(s)ds d\tau \\ & - 2\alpha V_{11}, \end{aligned} \quad (13)$$

$$\begin{aligned}
 V_{12}(x) &\leq h x^T(t) W x(t) - e^{-4\alpha h_2} \int_{-h_2}^t \int_{t+\tau}^t x^T(s) W x(s) ds d\tau \\
 &\quad - 2\alpha V_{12}, \\
 \dot{V}_{13}(x_t) &\leq h_2 \dot{x}^T(t) W_3 \dot{x}(t) - e^{-4\alpha h_2} \int_{-h_2}^0 \int_{t+\tau}^t \dot{x}^T(s) W_3 \dot{x}(s) ds d\tau \\
 &\quad - 2\alpha V_{13}, \\
 V_{14}(x) &\leq dx^T(t) Q x(t) - e^{-2\alpha d} \int_{t-d}^t x^T(s) Q x(s) ds \\
 &\quad - 2\alpha V_{14}.
 \end{aligned}$$

By Lemma 3 and Lemma 4, we have

$$\begin{aligned}
 &2f^T(x(t))B^T P x(t) - x^T(t)F^T P U^{-1} P F x(t) + x^T(t)B^T U B x(t), \\
 &2g^T(x(t-h(t)))C^T P x(t) \\
 &\leq x^T(t-h(t))G^T C^T U_3 C G x(t-h(t)) \\
 &\quad + x^T(t)P U^{-1} P x(t), \\
 &2w^T(t)E^T P x(t) \\
 &\leq \frac{\gamma}{2} w^T(t)w(t) + \frac{\gamma}{2} x^T(t)P E^T E P x(t), \\
 &2u^T(t-\tau(t))B^T P x(t) \leq \frac{e^{-2\alpha\tau}}{4} u^T(t-\tau(t))S_1 u(t-\tau(t)) \\
 &\quad + 4e^{2\alpha\tau} x^T(t)P_1 B_2 S_1^{-1} B_2^T P x(t),
 \end{aligned} \tag{14}$$

and the Leibniz-Newton formula gives

$$\begin{aligned}
 &-\tau e^{-2\alpha\tau} \int_{t-\tau}^t \dot{u}^T(s) S_2 \dot{u}(s) ds \\
 &\leq e^{-2\alpha\tau} x^T(t) P_1^{-1} B_1^T S_1 B_1 P_1^{-1} x(t) \\
 &\quad - \frac{e^{-2\alpha\tau}}{2} u^T(t-\tau(t)) S_1 u(t-\tau(t)).
 \end{aligned} \tag{15}$$

Denote

$$\sigma_1 = \int_{t-h_2}^{t-h(t)} x(s) ds, \sigma_2 = \int_{t-h(t)}^{t-h_1} x(s) ds. \tag{16}$$

Next, when  $0 \leq h_1 \leq h(t) \leq h_2$ , we have

$$\begin{aligned}
 \int_{t-h_2}^{t-h_1} x^T(s) Z_2 x(s) ds &= \int_{t-h_2}^{t-h(t)} \dot{x}^T(s) Z_2 \dot{x}(s) ds \\
 &\quad + \int_{t-h(t)}^{t-h_1} x^T(s) Z_2 x(s) ds.
 \end{aligned}$$

Using Lemma 4 gives

$$\begin{aligned}
 &h_{12} \int_{t-h_2}^{t-h_1} \dot{x}^T(s) Z_2 \dot{x}(s) ds \\
 &\geq \sigma_1^T(t) Z \sigma_1(t) + \frac{h(t)-h_1}{h(t)-h_1} \sigma_1^T(t) Z \sigma_1(t) \\
 &\quad + \sigma_2^T(t) Z \sigma_2(t) + \frac{h_2-h(t)}{h(t)-h_1} \sigma_2^T(t) Z \sigma_2(t).
 \end{aligned} \tag{17}$$

By reciprocally convex  $a = \frac{h-h(t)}{h_{12}}, b = \frac{h(t)-h_1}{h_{12}}$ , the

following inequality holds:

$$\begin{bmatrix} \sqrt{\frac{b}{a}} \sigma_1(t) \\ -\sqrt{\frac{a}{b}} \sigma_2(t) \end{bmatrix}^T \begin{bmatrix} Z_2 & Z_3 \\ * & Z_2 \end{bmatrix} \begin{bmatrix} \sqrt{\frac{b}{a}} \sigma_1(t) \\ -\sqrt{\frac{a}{b}} \sigma_2(t) \end{bmatrix} \geq 0, \tag{18}$$

which implies

$$\begin{aligned}
 &\frac{h(t)-h_1}{h_2-h(t)} \sigma_1^T(t) Z \sigma_1(t) + \frac{h_2-h(t)}{h(t)-h_1} \sigma_2^T(t) Z \sigma_2(t) \\
 &\geq \sigma_1^T(t) Z \sigma_1(t) + \sigma_2^T(t) Z \sigma_2(t).
 \end{aligned} \tag{19}$$

Then, we can get from (16)-(19) that

$$\begin{aligned}
 &h_{12} \int_{t-h_2}^{t-h_1} x^T(s) Z_2 x(s) ds \\
 &\geq \sigma_1^T(t) Z \sigma_1(t) + \sigma_2^T(t) Z \sigma_2(t) \\
 &\quad + \sigma_1^T(t) Z_3 \sigma_2(t) + \sigma_2^T(t) Z_3 \sigma_1(t).
 \end{aligned}$$

Thus

$$\begin{aligned}
 &-e^{-2\alpha h} h_{12} \int_{t-h_2}^{t-h_1} \dot{x}^T(s) Z_2 \dot{x}(s) ds \\
 &\geq -e^{-2\alpha h_2} [x(t-h(t)) - x(t-h_2)]^T Z_2 [x(t-h(t)) - x(t-h_2)] \\
 &\quad - e^{-2\alpha h_1} [x(t-h_1) - x(t-h(t))]^T Z_2 [x(t-h_1) - x(t-h(t))] \\
 &\quad - e^{-2\alpha h_2} [x(t-h(t)) - x(t-h_2)]^T Z_3 [x(t-h_1) - x(t-h(t))] \\
 &\quad - e^{-2\alpha h_1} [x(t-h_1) - x(t-h(t))]^T Z_3 [x(t-h(t)) - x(t-h_2)].
 \end{aligned} \tag{20}$$

By using Lemma 4 and Lemma 5 and the following identity relation:

$$\begin{aligned}
 0 &= -2\dot{x}^T R_1 \dot{x}(t) - 2x^T R_1 A x(t) + 2x^T R B f(x(t)) \\
 &\quad + 2\dot{x}^T R_1 C g(x(t-h(t))) + 2x^T R D \\
 &\quad \times \int_{t-d(t)}^t h(x(s)) ds + 2x^T R_1 E w(t) + 2x^T R_1 B_1 u(t) \\
 &\quad + 2\dot{x}^T R_1 B_2 u(t-\tau(t)) + 2x^T R_1 B_3 \int_{t-d_1(t)}^t u(s) ds.
 \end{aligned} \tag{21}$$

and from (14)-(21), we obtain

$$\begin{aligned}
 &V(t, x_t) + 2\alpha V(t, x_t) \\
 &\leq \gamma w^T(t)w(t) + \xi^T(t) M_1 \xi(t) + \dot{x}^T(t) M_2 \dot{x}(t) \\
 &\quad + x^T(t) M_3 x(t) + x^T(t) M_4 x(t) - x^T(t) [A_1^T A_1 \\
 &\quad + A_1^T C_1 B_1 P_1^{-1} + P_1^{-1} B_1^T C_1^T A_1 + P_1^{-1} B_1^T C_1^T B_1 P_1^{-1}] x(t) \\
 &\quad - 2x^T(t) [A_1^T B_1 + P_1^{-1} B_1^T C_1^T B_1] x(t-h(t)) \\
 &\quad - 2x^T(t) [A_1^T D_1 + P_1^{-1} B_1^T C_1^T D_1] \int_{t-d}^t x(s) ds \\
 &\quad - 2x^T(t) [A_1^T E_1 + P_1^{-1} B_1^T C_1^T E_1] w(t) \\
 &\quad - x^T(t-h(t)) B_4^T B_4 x(t-h(t)) \\
 &\quad - 2x^T(t-h(t)) \bar{B}^T \bar{D} \int_{t-d}^t x(s) ds \\
 &\quad - \left( \int_{t-d}^t x(s) ds \right)^T B_4^T E_4 w(t) \\
 &\quad - \left( \int_{t-d}^t x(s) ds \right)^T D_1^T D_1 \int_{t-d}^t x(s) ds \\
 &\quad - 2 \left( \int_{t-d}^t x(s) ds \right)^T D_1^T E_1 w(t) - w^T(t) E_1^T E_1 w(t),
 \end{aligned} \tag{22}$$

where

$$M_1 = \Pi + F^T P_1 U_2^{-1} P_1 F + P_1 U_3^{-1} P_1 + 2de^{2\alpha d} PDU^{-1} D^T P_1 + 4e^{2\alpha\tau} P B S^{-1} B^T P + 2d e^{2\alpha d_1} P B S^{-1} B^T P + \frac{2}{\gamma} P E^T E P,$$

$$M_2 = -0.4R_1 + R_1 B U_2^{-1} B R_1 + R_1 C U_3^{-1} C^T R_1 + 2de^{2\alpha d} R D U^{-1} D^T R + \frac{2}{\gamma} R E^T E R$$

$$+ 4e^{2\alpha\tau} R_1 B_2 S_1^{-1} B_2^T R_1 + R_1 B_1 S_3^{-1} B_1^T R_1 + 2d_1 e^{2\alpha d_1} R_1 B_3 S_2^{-1} B_3^T R_1, \\ M_3 = -0.5e^{-2\alpha\tau} Q + dP^{-1} B^T S B P^{-1} + e^{-2\alpha d_1} P^{-1} B^T S B P^{-1} + P^{-1} B^T S B P^{-1},$$

$$M_4 = -0.1R_1 + \tau^2 P_1^{-1} B_1^T S_1 B_1 P_1^{-1}, \\ \xi^T(t) = [x^T(t) \ x^T(t-h_1) \ x^T(t-h_2) \ x^T(t-h_3)] \\ \int_{t-h_1}^t x^T(s) ds \int_{t-h_2}^t x^T(s) ds \int_{t-h_3}^t x^T(s) ds \\ \int_{t-d}^t x(s) ds \int_{-h_2}^t \int_{t+s} x(\theta) d\theta ds \dot{x}(t)].$$

Using Schur complement lemma, pre-multiplying and post-multiplying  $M_1, M_2, M_3$  and  $M_4$  by  $P_1$  and  $P_1$  respectively, the inequality  $M_1, M_2, M_3$  and  $M_4$  are equivalent to  $\sum_1 < 0, \sum_2 < 0, \sum_3 < 0$  and  $\sum_4 < 0$  respectively.

Letting  $w(t) = 0$ , we obtain from the inequality (22) we have,

$$\dot{V}(t, x_t) \leq -2\alpha V(t, x_t), \quad \forall t \geq 0, \quad (23)$$

Integrating both sides of (23) from 0 to  $t$ , we obtain

$$V(t, x_t) \leq V(0, x_0) e^{-2\alpha t}, \quad \forall t \geq 0,$$

Taking the condition (12) into account, we have

$$\lambda_1 \|x(t)\|^2 \leq V(t, x_t), \quad V(0, x_0) e^{-2\alpha t} \leq \lambda_2 \|\phi\|_c^2 e^{-2\alpha t}.$$

Then, the solution  $\|x(t, \phi)\|$  of the system (3) satisfy

$$\|x(t, \phi)\| \leq \sqrt{\frac{\lambda_2}{\lambda_1}} \|\phi\|_c e^{-\alpha t}, \quad \forall t \geq 0,$$

which implies that the zero solution of the closed-loop system is  $\alpha$ -stable. To complete the proof of the theorem, it remains to show the  $\gamma$ -optimal level condition (ii). For this, we consider the following relation:

$$\int_0^t [\|z(s)\|^2 - \gamma \|w(s)\|^2] ds \leq \int_0^t [\|z(s)\|^2 - \gamma \|w(s)\|^2 + V(s, x)] ds + V(0, x). \quad (24)$$

Hence, from (24) it follows that

$$\int_0^t \|z(s)\|^2 ds \leq \int_0^t \gamma \|w(s)\|^2 ds + \lambda_2 \|\phi\|_c^2.$$

Letting  $t \rightarrow \infty$ , and setting  $c_0 = \frac{\lambda_2}{\gamma}$ , we obtain that

$$\frac{\int_0^\infty \|z(t)\|^2 dt}{c_0 \|\phi\|_c^2 + \int_0^\infty \|w(t)\|^2 dt} \leq \gamma,$$

for all non-zero  $w(\cdot) \in L_2([0, \infty), R^n)$ ,

$\phi(\cdot) \in C[-\rho, 0], R^n$ . This completes the proof of the theorem.

#### IV. NUMERICAL EXAMPLES

In this section, numerical example is given to present the effectiveness and applicability of our stability results.

**Example 8.** Consider neural network (3) with parameters as follows:

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, B = \begin{bmatrix} -0.7 & 0.2 \\ 0.4 & -0.1 \end{bmatrix}, C = \begin{bmatrix} 0.7 & -0.8 \\ 0.5 & -0.9 \end{bmatrix}, \\ D = \begin{bmatrix} 0.7 & -0.7 \\ -0.1 & -0.4 \end{bmatrix}, E = \begin{bmatrix} -0.1 & 0.2 \\ 0 & 0.4 \end{bmatrix}, F = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.3 \end{bmatrix}, \\ G = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix}, H = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.2 \end{bmatrix}, A^1 = \begin{bmatrix} -0.2 & 0.3 \\ 0 & -0.4 \end{bmatrix}, \\ B^1 = \begin{bmatrix} -0.4 & 0 \\ 0 & -0.4 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, B_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ B^2 = \begin{bmatrix} -0.4 & 0 \\ -0.1 & -0.5 \end{bmatrix}, C^1 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & -0.5 \end{bmatrix}, D^1 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & -0.4 \end{bmatrix}, \\ E_1 = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.3 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, h(\cdot) = \tanh(\cdot) \\ f(\cdot) = g(\cdot) = \begin{bmatrix} |x_1(t)+1| - |x_1(t)-1| \\ |x(t)+1| - |x(t)-1| \end{bmatrix}.$$

From the conditions (8)-(11) of Theorem 7, we let  $\alpha = 0.01, h_1 = 0.1, h_2 = 0.3, d = 0.3, d_1 = 0.5$ , and  $\tau = 0.4$ . By using the LMI Toolbox in MATLAB, we obtain  $\gamma = 1.7637$ ,

The feedback control is given by

$$u(t) = B_1 P_1^{-1} x(t) = \begin{bmatrix} -0.4478 & -0.0894 \\ -0.0894 & -0.5228 \end{bmatrix}, \quad t \geq 0.$$

Moreover, the solution  $x(t, \phi)$  of the system satisfies

$$\|x(t, \phi)\| \leq 1.2300 e^{-0.01t} \|\phi\|_c.$$

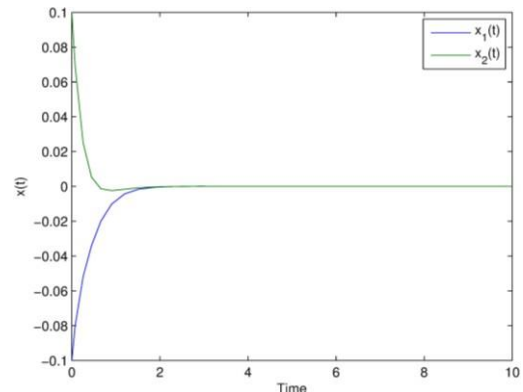


Fig. 1. Response solution of the system (3) where  $w(t) = 0$ .

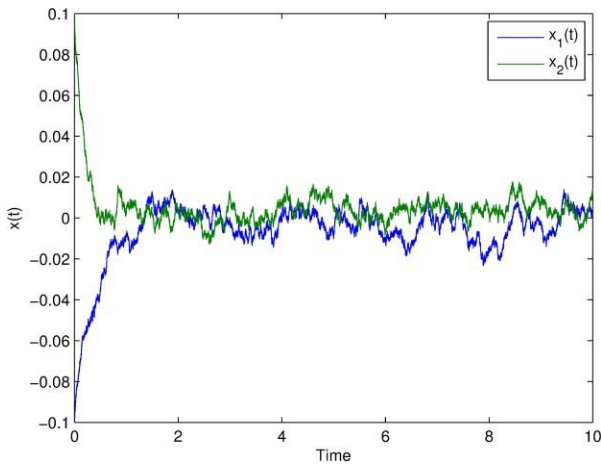


Fig. 2. Response solution of the system (3).

Fig. 1. shows the response solution  $x(t)$  of the neural network system (3) where  $w(t)=0$  and the initial condition  $\phi(t)=[-0.1 \ 0.1]^T$ .

Fig. 2. shows the response solution  $x(t)$  of the neural network system (3) where  $w(t)$  is Gaussian noise with mean 0 and variance 1 and the initial condition  $\phi(t)=[-0.1 \ 0.1]^T$ .

## V. CONCLUSION

In this paper, the problem of an  $H_\infty$  control for neural network systems with interval and distributed time-varying delays was investigated. It is assumed that the interval and distributed time-varying delays are not necessary to be differentiable. Firstly, we considered an  $H_\infty$  control for exponential stability of neural network with interval and distributed time-varying delays via hybrid feedback control. Secondly, by using a novel Lyapunov-Karsovskii functional, the employment of a tighter bounding technique, some slack matrices and newly introduced convex combination condition in the calculation, improved delay-dependent sufficient conditions for the  $H_\infty$  control with exponential stability of the system are obtained. Finally, a numerical example has been given to illustrate the effectiveness of the proposed method. The results in this paper improve the corresponding results of the recent works.

## REFERENCE

- [1] M. S. Alwan, X. Z. Liu, and W. C. Xie, "On design of robust reliable  $H_\infty$  control and input-to-state stabilization of uncertain stochastic systems with state delay," *Commun. Nonlinear Sci. Numer. Simulat.*, vol. 18, pp. 1047–1056, 2013.
- [2] J. Gao, B. Huang, Z. Wang, and D. G. Fisher, "Robust reliable control for a class of uncertain nonlinear systems with time-varying multi-state time delays," *Int. J. Syst. Sci.*, vol. 32, pp. 817–824, 2001.
- [3] Z. Wang, B. Huang, and H. Unbehauen, "Robust reliable control for a class of uncertain nonlinear state-delayed systems," *Automatica.*, vol. 35, pp. 955–963, 1999.
- [4] Z. G. Xiang and Q. W. Chen, "Robust reliable control for uncertain switched nonlinear systems with time delay under asynchronous switching," *Appl. Math. Comput.*, vol. 216, pp. 800–811, 2010.

- [5] S. Haykin, *Neural Networks*, Prentice-Hall, Englewood Cliffs, New Jersey, 1994.
- [6] X. Yang, Q. Song, J. Liang, and B. He, "Finitetime synchronization of coupled discontinuous neural networks with mixed delays and nonidentical perturbations," *J. Franklin Inst.*, vol. 352, pp. 4382–4406, 2015.
- [7] H. Zhang and Z. Wang, "New delay-dependent criterion for the stability of recurrent neural networks with time-varying delay," *Sci. China Inf. Sci.*, vol. 52, pp. 942–948, 2009.
- [8] X. Li and C. E. D. Souza, "Delay-dependent robust stability and stabilization of uncertain linear delay systems: A linear matrix inequality approach," *IEEE Trans. Automat. Control*, vol. 42, pp. 1144–1148, 1997.
- [9] M. Syed Ali, "On exponential stability of neutral delay differential system with nonlinear uncertainties," *Commun. Nonlinear Sci. Numer. Simul.*, vol. 17, pp. 2595–2601, 2012.
- [10] M. Syed Ali and P. Balasubramaniam, "Exponential stability of time delay systems with nonlinear uncertainties," *Int. J. Comput. Math.*, vol. 87, pp. 1363–1373, 2010.
- [11] N. T. Thanh and V. N. Phat, " $H_\infty$  control for nonlinear systems with interval non-differentiable time-varying delay," *Eur. J. Control*, vol. 19, pp. 190–198, 2013.
- [12] Y. Du, X. Liu, and S. Zhong, "Robust reliable  $H_\infty$  control for neural networks with mixed time delays," *Chaos Solitons Fractals*, vol. 91, pp. 1–8, 2016.
- [13] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay System*, Birkhauser, Boston, Massachusetts, 2003.



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