Improved ICI Self Cancellation Scheme for Phase Rotation Error Reduction in OFDM System

Miftahur Rahman, Preyom Kanti Dey, and M. F. Rabbi Ur Rashid

***Abstract*—Orthogonal frequency division multiplexing (OFDM) is sensitive to carrier frequency offset (CFO).This CFO mainly occurs because of the frequency mismatch in the transmitter and the receiver. As a result, the orthogonality between the subcarriers are destroyed causing not only the inter carrier interference (ICI) in a sub carrier but also a phase rotation in the received complex signal. Due to the phase rotation of the received data the receiver decode the data incorrectly.Several methods like self cancellation, modified self cancellation, pulse shaping, maximum Likelihood offset detection are developed to overcome the ICI problem. Self cancellation techniques are based on algorithms making use of symmetry and conjugate properties of the sub-carriers [1,2]. But none of these methods ensure the reduction of phase rotation problem. In this paper we have proposed a self cancellation method which uses for sub carriers for a particular data transmission to reduce the Phase Rotation of the received signal. We have compared the phase rotation of the received signal with respect to the Carrier Frequency Offset (CFO) of our proposed scheme with the ICI Self Cancellation Schemes. From our simulation, we find that our proposed scheme provides less rotation to the received data for a particular CFO than any other schemes.**

***Index Terms*—OFDM, Inter carrier interference, phase rotation error, carrier frequency offset**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is being considered for many emerging wireless applications. It has been accepted for several wireless LAN standards as well as mobile multimedia applications [3]. One major difficulty, however, is its sensitivity to frequency offset caused by misalignment in carrier frequencies or the Doppler shift. These imperfections will destroy sub-carrier orthogonality and introduce inter carrier interference (ICI) among subcarriers in addition to attenuation and rotation of each subcarrier [4].Several methods have been developed so far to minimize or cancel the effect of frequency offset. One of the effective methods is ICI self cancellation method. In conventional OFDM system each sub carrier carries a single data. So interference comes from all other sub carriers.

In self cancellation technique [5], the same data(“a”) is transmitted in two adjacent sub carriers n and n+1. The first one is weighted by '1' and the second one is by '−1' . It is assumed that, the two adjacent sub carriers will have same inter carrier interference effect. So, if we deduct the received signal Y(n+1) from Y(n) the inter carrier interference effect will be nullified.

In the modified self cancellation scheme [6], it is considered that, the ICI effects on nth sub carrier and (N −1− n)th sub carriers are same, where N is the total number of sub carriers. So, data 'a' is transmitted in nth and data '−a' in (N-1-n)th. Or in other words nth data is weighted by '1' and (N −1− n)th data is weighted by '−1' . In the receiver end, received signal Y(N-1-n) is deducted from Y(n).

One constrain of these schemes is that, the probability of the phase rotation error in the received signal created by carrier frequency offset is almost equal to the basic OFDM system.

In the proposed work, an algorithm has been proposed for minimizing the phase rotation error. In this algorithm, the derivation of Phase rotation error has been carried out in two steps. In the first step, mathematical derivation of the Phase Rotation error has been presented. In the second step, numerical calculation using MATLAB has been implemented. to show the comparison of Phase Rotation Error between the proposed and the existing schemes.

II. Phase Rotation of Conventional OFDM Systems

In a conventional OFDM system normalized carrier frequency offset (CFO), is being considered. Then the received signal on sub-carrier **n** can be written as

$$Y(n) = X(n)S(0) + \sum_{\substack{\ell=0 \\ \ell \neq n}}^{N-1} X(\ell)S(\ell - n) + w_n \quad (1)$$

where $n = 0,1,\cdots,N\text{-}1$.

In (1) X(n) is the transmitted symbol for the n^{th} sub-carrier, $\{X(\ell),\ \ell = 0,1,\cdots,N\text{-}1\}$ are independent equally probable QPSK symbols, w_n is the additive noise sample and N is the total number of sub-carriers.

The first term in the right-hand side of (1) represents the desired signal. The second term is the ICI components. The sequence $S(\ell - n)$ is defined as the ICI coefficient between ℓ^{th} and n^{th} sub-carriers. So the phase rotation for basic OFDM system is

$$\theta = \tan^{-1}\left(\frac{\mathrm{Im}[S(0)]}{\mathrm{Re}[S(0)]}\right) \quad (2)$$

In the case of ICI self cancellation scheme the received signal is

Manuscript received January 5, 2012; revised February 28, 2012.
The authors are with the North South University, Dhaka, Bangladesh (e-mail: mrahman@northsouth.edu; e-mail: preyomdey@yahoo.com; e-mail: u_rabbi@yahoo.com).

$$Y(n) = X(n)[-S(-1) + 2*S(0) + S(1)] + \sum_{\substack{l=0 \\ \ell \neq even \\ \ell \neq n}}^{N-2} X(\ell)\left[-S(\ell-n-1) + 2*S(\ell-n) - S(\ell-n+1)\right] + \{w(n) - w(n+1)\} \quad (3)$$

We can see that, for n=0 and N=64 the first term is the original signal. The message signal X(0) is multiplied by $|[-S(-1) + 2*S(0) + S(1)]$ This coefficient gives a rotation to the X(0) and creates the phase rotation error. If the rotation is more than 45^0 in QPSK modulation the bit is considered as out of the decision region. So the phase rotation for this scheme is

$$\theta = \tan^{-1}\left(\frac{\text{Im}[|[-S(-1) + 2*S(0) + S(1)]]}{\text{Re}[|[-S(-1) + 2*S(0) + S(1)]]}\right) \quad (4)$$

Similarly, for modified self cancellation scheme the phase rotation is

$$\theta = \tan^{-1}\left\{\frac{\text{Im}([2S(0) - S(63) - S(-63)])}{\text{Re}([2S(0) - S(63) - S(-63)])}\right\} \quad (5)$$

III. Proposed Algorithm

Here, instead of two sub-carriers for a single data, four sub-carriers have been used. The data allocation for the K^{th} sub-carrier is *{X(n),* $e^{-j\frac{\pi}{2}}$ *X(n), - X(n),* $e^{j\frac{\pi}{2}}$ *X(n)}*, *n*=0,4,…..*N- 4*. Here,N is the total number of sub-carriers. That means, the same data is shifted by different phase in different sub carriers. For example, if in 0^{th} carrier the complex data is 'a', then in 1^{st} subcarrier contains 'a$e^{-j\frac{\pi}{2}}$',the next two contains '-a' and 'a$e^{j\frac{\pi}{2}}$' respectively. Suppose, 'a' is a complex data and it is 1+1j. So data in four consecutive carriers will be like 1+1j, 1-1j, -1-1j, -1+1j respectively. As the received signal is the summation of desired signal and ICI components, so the received signal at n^{th} sub carrier Y(n) is like,

$$\begin{aligned} Y(n) &= \sum_{\ell=0}^{N-1} X(\ell)S(\ell-n) + w(n) \\ &= X(n)S(0) + X(n+1)S(1) + X(n+2)S(2) + X(n+3)S(3) + \sum_{\substack{\ell=0 \\ \ell\neq n,n+1, \\ n+2,n+3}}^{N-1} X(\ell)S(\ell-n) + w(n) \\ &= X(n)S(0) + e^{-j\frac{\pi}{2}}X(n)S(1) - X(n)S(2) + e^{j\frac{\pi}{2}}X(n)S(3) + \sum_{\substack{\ell=0 \\ \ell\neq n,n+1, \\ n+2,n+3}}^{N-1} X(\ell)S(\ell-n) + w(n) \\ &= X(n)[S(0) + e^{-j\frac{\pi}{2}}S(1) - S(2) + e^{j\frac{\pi}{2}}S(3)] + \sum_{\substack{\ell=0 \\ \ell=l+4 \\ \ell\neq n}}^{N-4} X(\ell)[S(\ell-n) + e^{-j\frac{\pi}{2}}S(\ell-n+1) - S(\ell-n+2) + e^{j\frac{\pi}{2}}S(\ell-n+3)] + w(n) \end{aligned} \quad (6)$$

For $(n+1)^{th}$,$(n+2)^{th}$, $(n+3)^{th}$ carrier received signal will be

$$Y(n+1) = \sum_{\substack{l=0 \\ l=l+4 \\ l\neq n}}^{N-4} X(l)[S(l-n-1) + e^{-j\frac{\pi}{2}}S(l-n) - S(l-n+1) + e^{j\frac{\pi}{2}}S(l-n+2)] + w(n+1) \quad (7)$$

$$Y(n+2) = \sum_{\substack{l=0 \\ l=l+4 \\ l\neq n}}^{N-4} X(l)[S(l-n-2) + e^{-j\frac{\pi}{2}}S(l-n-1) - S(l-n) + e^{j\frac{\pi}{2}}S(l-n+1)] + w(n+2) \quad (8)$$

$$Y(n+3) = \sum_{\substack{l=0 \\ l=l+4 \\ l\neq n}}^{N-4} X(l)[S(l-n-3) + e^{-j\frac{\pi}{2}}S(l-n-2) - S(l-n-1) + e^{j\frac{\pi}{2}}S(l-n)] + w(n+3) \quad (9)$$

The received signal is then decoded in the following manner to decode the received signal in the n^{th} sub-carrier,

$$\begin{aligned} Y`(n) &= Y(n) - e^{-j\frac{\pi}{2}}Y(n+1) - Y(n+2) + e^{-j\frac{\pi}{2}}Y(n+3) \\ &= \sum_{\substack{l=0 \\ l=l+4}}^{N-4} X(l)[[3*S(l-n) - 2*S(l-n+2) - S(l-n-2)] + e^{-j\frac{\pi}{2}}[2*S(l-n+1) - 3*S(l-n-1) + S(l-n-3)] + \\ &\quad e^{j\frac{\pi}{2}}[S(l-n+3) - S(l-n+1)] + e^{-j\pi}[-S(l-n) + S(l-n-2)]] + \{w(n) - w(n+1) - w(n+2) + w(n+3)\} \\ &= X(n)[[3*S(0) - 2*S(2) - S(-2)] + e^{-j\frac{\pi}{2}}[2*S(1) - 3*S(-1) + S(-3)] + e^{j\frac{\pi}{2}}[S(3) - S(1)] + e^{-j\pi}[-S(0) + S(-2)]] + \\ &\quad \sum_{\substack{l=0 \\ l=l+4 \\ l\neq K}}^{N-4} X(l)[[3*S(l-n) - 2*S(l-n+2) - S(l-n-2)] + e^{-j\frac{\pi}{2}}[2*S(l-n+1) - 3*S(l-n-1) + S(l-n-3)] + \\ &\quad e^{j\frac{\pi}{2}}[S(l-n+3) - S(l-n+1)] + e^{-j\pi}[-S(l-n) + S(l-n-2)]] + \{w(n) - w(n+1) - w(n+2) + w(n+3)\} \end{aligned} \quad (10)$$

Here, the first term is the message signal and its coefficient. Second term is the ICI component and its coefficient. So for n^{th} carrier the ICI coefficient term is

$$S`(l-n)=\sum_{\substack{l=0\\l=l+4\\l\neq n}}^{N-4}[[3*S(l-n)-2*S(l-n+2)-$$

$$S(l-n-2)]+e^{-j\frac{\pi}{2}}[2*S(l-n+1)-3*S(l-n-1)+S(l-n-3)]+e^{j\frac{\pi}{2}}[S(l-n+3)-S(l-n+1)]+e^{-j\pi}[-S(l-n)+S(l-n-2)]] \quad (11)$$

Finally for n=0th sub carrier the ICI coefficient term is

$$S`(l-0)=[[3*S(0)-2*S(2)-S(-2)]+e^{-j\frac{\pi}{2}}[2*S(1)-3*S(-1)+S(-3)]+e^{j\frac{\pi}{2}}[S(3)-S(1)]+e^{-j\pi}[-S(0)+S(-2)]] \quad (12)$$

As discussed earlier, any signal X(n) is rotated by its coefficient term. In this proposed scheme the signal is rotated by the term[3*S(0)-2*S(2)-S(-2)+ $e^{-j\frac{\pi}{2}}$ {2*S(1)-3*S(-1)+S(-3)}+ $e^{j\frac{\pi}{2}}$ {S(3)-S(1)}+ $e^{-j\pi}$ {-S(0)+S(-2)}].Suppose , for QPSK modulation if this phase shift is greater than 45^0 then that message will be logically corrupted. So the phase rotation is

$$\theta=\tan^{-1}\left\{\frac{\begin{array}{l}\mathrm{Im}(3*S(0)-2*S(2)-S(-2)+\\\{2*S(1)-3*S(-1)+S(-3)\}+\\\{S(3)-S(1)\}+\{-S(0)+S(-2)\})\end{array}}{\begin{array}{l}\mathrm{Re}(3*S(0)-2*S(2)-S(-2)+\\\{2*S(1)-3*S(-1)+S(-3)\}+\\\{S(3)-S(1)\}+\{-S(0)+S(-2)\})\end{array}}\right\} \quad (13)$$

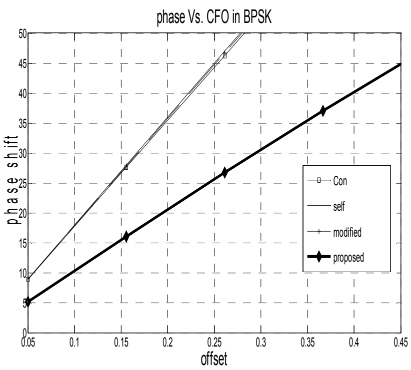

Fig. 1. Phase shift vs. normalized frequency offset

IV. Result and Discussion

Phase shift in the modulation schemes changes according to the change of the frequency offset. If the phase shift is significant enough for a particular modulation scheme, than the original data may be corrupted. For example, in case of QPSK, if the phase shift is more than 45°, the original bits might change.

The graph above clearly shows the significance of proposed algorithm. For all the other cancellation schemes, we can see that if the frequency offset is 0.25, the phase shift is about 45°, which can be a serious problem if the modulation scheme is chosen as QPSK. Instead the proposed algorithm shows a better performance.

The curve for this scheme is more flat and the phase shift shows better immune to the value of ε. Hence the same frequency shift 45° will occur when the frequency offset is 0.45.

V. Conclusion

In this paper, we have suggested a simple ICI cancellation algorithm to reduce the phase rotation error of the OFDM system by using four sub-carriers for single data transmission. Form the simulation result of phase rotation comparison , the proposed algorithms have shown significant improvement than the existing schemes. In our proposed algorithms, only QPSK modulation scheme has been considered. Further comparison could be done based on other modulation schemes such as QAM, BPSK etc.

Acknowledgment

First of all, we would like to express our gratitude to the Almighty. Then, we would like to mention our families' dedication and sacrifice to this project, without their support it was really impossible to finish this work. At last, we would also like to thank to our friends for their excellent support and encouragement.

References

[1] H.-G. Yeh, Yuan-Kwei Chang, and Babak Hassibi, "AScheme for Cancelling Intercarrier Interference using Conjugate Transmission in Multicarrier Communication Systems," *IEEE Transactions on Wireless Communications*, vol. 6, no. 1, January 2007.

[2] H.-G. Yeh and Y.-K. Chang, "A Conjugate Operation for Mitigating Intercarrier Interference of OFDM Systems," *IEEE Xplore,* 0-7803-8521-7/04/$20.00 © 2004 IEEE.

[3] V. K. Dwivedi and G. Singh, "*An Efficient BER Analysis of OFDM Systems with ICI Conjugate Cancellation Method*," *PIERS Proceedings*, Cambridge, USA, July 2-6, 2008.

[4] K. Sathananthan, R. M. A. P. Rajatheva, and Slimane Ben Slimane, "Analysis of OFDM in the Presence of Frequency Offset and a Methosd to Reduce Performance Degradation," *IEEE Xplore,* 0-7803-6451-1/00/$10.00@2000 IEEE.

[5] Z. Zhu, X. Tang, J. Zuo, "Self-Cancellation Method of OFDM ICI," *IEEE Xplore,* 978-1-4244-2108-4/08/$25.00 © 2008 IEEE.

[6] A. R. Zolghadr Asli, B. Amin Shah, M. A. Masnadi, Shirazi, M. H. Ghamat, "Comparison of Some Methods Including Use of Coding for Reduction of ICI in OFDM Channels," *Iranian Journal of Information Science and Technology*, vol. 4, no. 2, pp. 1-14, July/December, 2006.